

# Pricing and investment of cross-border transport infrastructure

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## 1 Introduction

Cross-border transport infrastructure plays an important role of supporting trade with neighboring countries, which account for significant share of total trade. Moreover, recent wave of regional integration would leads to greater needs for investment in transport infrastructure serving the region, since well-developed transport infrastructure is essential for successful regional integration. In fact, Trans-European Transport Network, the Asian Land Transport Infrastructure Development (ALTID) project are considered as initiatives to facilitate regional integration.

Mun and Nakagawa (2007) discuss the problem of resource allocation concerning the provision of cross-border transport infrastructure connecting two neighboring countries. An investment in the infrastructure on one country decreases the transportation costs of both import and export goods, which benefits not only the home country but also the other country. In this case, independent decision making leads to under-investment of infrastructure, since investment decision of each country does not take into account the benefit of other country. Mun and Nakagawa focus on the role of foreign aid to improve the efficiency, and show that the aid may make not only the recipient but also the donor better off. The limitation of this paper is that it focuses only on investment decisions assuming that the price for using the infrastructure is free.

Although free access to transport infrastructures (such as public road) is widely applied, there exist some types of transportation modes that impose the fees for the use of infrastructure (fare for rail). At the same time, there has been an increasing tendency of tolling on public roads, for several reasons such as congestion management, funds for road investments, etc. (Glaister and Graham (2004)). For example, the truck tolling system was recently introduced in Germany from 2005.

The present paper extends our earlier work by including pricing policies. We develop a simple two-country model of international trade where the transportation cost between two countries depends on capacity and price (e.g., road toll, rail fare) of infrastructure.

The government of each country chooses capacity and price of infrastructure use within its territory so as to maximize the welfare of its citizen. The government faces the following trade-off: investment in capacity would lower the transport cost thereby increase the gains from trade, but increase the fiscal burden; raising the price would increase the revenue but decrease the gains from trade.

The earlier result of under-investment in the case of free access may be modified if we incorporate pricing; the decision rule of choosing the capacity turns to efficient one. Investment induces increase in the volume of traded goods, which raises not only the gains from trade but also revenue from pricing. The latter effect plays a role of giving the governments incentives of more investment. However, pricing rule is inefficient in that each government charges an excessively high price for infrastructure uses. Thus, despite the efficient investment rule, the resulting capacity does not attain the optimal level.

We evaluate two regimes, with and without pricing, in terms of the efficiency in provision of the infrastructure, welfare of the two countries, and global welfare of two-country economy as a whole. Furthermore, considering the fact that a growing number of toll roads have been constructed and operated privately or by various forms of public-private partnerships (Roth (1996)), we examine the alternative systems of providing cross-border transport infrastructure, such as private operation, control of investment decision of private firm by design of bidding for franchise.<sup>1</sup>

It is only recently that economists are interested in pricing and investment decisions of transport infrastructure by multiple governments (Bond (2006), De Borger, Dunkerley, Proost (2007), Fukuyama (2006), Levinson (2000)).

Bond (2000) investigated the consequences of independent decision making by governments concerning infrastructure investment, and examined the effects of trade liberalization on the incentive to invest. Fukuyama (2005) also discusses a similar problem by means of numerical simulations. These papers focus only on investment decisions. Levinson (2000) looks at strategic interactions between governments on a serial network. Levinson keeps capacity fixed, and focuses on the choice of revenue raising mechanisms (tax versus toll). He finds that larger regions are more likely to tax than smaller regions.

Our paper is closely related to the recent work by De Borger, Dunkerley and Proost (2007) that deals with pricing and investment in the setting of two transport links in series, each of which is controlled by a different government. Transport links are congestible and traffic on each link consists of local trips and transit. Transit trips are neither originated from nor destined to one of two regions, while trips between neighboring regions are assumed to be zero. De Borger, Dunkerley and Proost (2007) evaluate numerically the effects of toll discrimination between local and transit trips. Our paper considers trips between neighboring regions that would be quantitatively more important than transit traffic in many cases. We evaluate a larger set of alternative regimes such as private operation, etc., and obtain the ranking of different regimes analytically in the case that two countries are symmetric.

The rest of the paper is organized as follows. Section 2 describes the model of

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<sup>1</sup>de Palma and Lindsey (2000) analyze road pricing in the case that roads are operate privately. Yang, H., and Meng, Q. (2000) investigate the effect of new road project by BOT in the setting of network with route choice. Verhoef (2007) evaluates alternative highway franchise regimes for different network structure, such as parallel and serial. Primary interests of these works are on the role of pricing to control traffic congestion, and they do not deal with the situation that multiple governments are involved in pricing and investment of transport infrastructure.

two country economy. Section 3 presents alternative regimes concerning pricing and investment of cross-border transport infrastructure: free-access, pricing and investment by national government, private operation, and user cost minimization. These alternative regimes are evaluated analytically and numerically in Section 4. Section 5 concludes the paper.

## 2 The Model

### 2.1 The setting

Consider an economy with two countries producing three goods. Each country is indexed by  $i$  ( $i = 1, 2$ ). All households in the same country have identical preferences and labor skills. There are  $l_i$  households in country  $i$ . The two countries may be different in country size, income, and preferences.

The three goods are indexed by 1, 2, and  $z$ . The productions of goods 1 and 2 are completely specialized, i.e., country  $i$  produces good  $i$ . Each household in the economy consumes all three goods. Thus, country 1 exports goods 1 and imports good 2 and vice versa. Unlike goods 1 and 2, good  $z$  is produced in both countries, which is set as the numeraire. Labor is the only input for the production of the three goods. Goods and factor markets are perfectly competitive.

When goods 1 and 2 are traded, transportation costs are incurred. On the other hand, good  $z$  is transported without cost. The transportation costs of goods 1 and 2 depend on capacity and price (e.g., road toll, rail fare) of infrastructure. We assume that the transport infrastructure is produced from good  $z$  with a constant returns to scale technology.

### 2.2 Consumption

The preferences of a household in country  $i$  are represented by the following utility function

$$u_i(x_i^i, x_i^j, z_i),$$

where  $x_i^i$ ,  $x_i^j$ , and  $z_i$  are respectively the consumption of goods  $i$ ,  $j$ , and  $z$ . We assume that  $u_i$  is twice continuously differentiable. Each household is endowed with one unit of labor and levied a poll tax. The household's disposable income is defined as

$$w_i^g = w_i - \tau_i,$$

where  $w_i$  is the wage rate in country and  $\tau_i$  is the poll tax. The budget constraint is given by

$$w_i^g = z_i + p_i^i x_i^i + p_i^j x_i^j,$$

where  $p_i^i$  and  $p_i^j$  represent the prices of goods  $i$  and  $j$  in country  $i$ . We suppose that goods 1 and 2 are normal goods.

Solving the utility maximization problem, we get the household's demand functions as

$$x_i^i(p_i^i, p_i^j, w_i^g), x_i^j(p_i^i, p_i^j, w_i^g), z_i(p_i^i, p_i^j, w_i^g),$$

and the indirect utility function

$$v_i(p_i^i, p_i^j, w_i^g).$$

### 2.3 Production

Each of the three goods is produced with a linear production technology. The production of good  $i$  in country  $i$  requires  $a_i^i$  amounts of labor. It follows that

$$p_i^i = w_i a_i^i, \text{ for } i = 1, 2.$$

Since the good  $z$  is set as the numeraire, the following relation should hold

$$1 = w_i a_i^z, \text{ for } i = 1, 2,$$

where  $a_i^z$  is the amount of labor required to produce one unit of good  $z$  in country  $i$ .

### 2.4 Transportation and trade

We suppose that there is a single location in each country at which goods are produced and consumed. We call this location as a market. Traded goods are transported between two markets. Labor is only input for production of transportation service. The transport cost from the market in country  $i$  to the border,  $c_i$ , is defined as

$$c_i = f_i + w_i t_i,$$

where  $f_i$  is the price of infrastructure use within country  $i$  and  $t_i$  is the amount of labor required for transportation. We interpret  $t_i$  as the transport time from the market in country  $i$  to the border of the two countries.  $t_i$  depends on the capacity of transport infrastructure in country  $i$ , namely,

$$t_i = t_i(k_i).$$

We assume that an investment in transport infrastructure increases capacity, thereby saves the labor (or time) required for transportation and that the investment is decreasing return to scale:  $t_i' \equiv dt_i/dk_i < 0$ ,  $t_i'' \equiv d^2t_i/dk_i^2 > 0$ .

The transport cost between two countries is given by  $c_1 + c_2$ . We assume perfect competition among trading firms, which eliminates positive profits. The prices of the traded goods satisfy

$$p_i^j = p_j^j + c_1 + c_2, \text{ for } i, j = 1, 2, j \neq i. \quad (1)$$

## 3 Capacity and price of transport infrastructure

In this section, we consider the following five regimes: the first-best optimum, free access regime, pricing regime, private operation regime, and user cost minimization regime.

### 3.1 The first-best optimum (Regime O)

In this paper, the first-best optimum is characterized as the solution to a global welfare maximization problem, which is called as regime O. Social planner chooses the price of infrastructure use, the capacity of infrastructure, and the poll tax. The problem to be solved is stated as follows

$$\max_{f_1, f_2, k_1, k_2, \tau_1, \tau_2, u_1, u_2} W(u_1, u_2)$$

subject to

$$u_i = v_i \left( p_i^i, p_j^j + f_1 + f_2 + w_1 t_1 + w_2 t_2, w_i - \tau_i \right), \quad j \neq i$$

$$p_1^k k_1 + p_2^k k_2 = l_1 \tau_1 + l_2 \tau_2 + (f_1 + f_2) (l_1 x_1^2 + l_2 x_2^1),$$

where  $W(\cdot)$  is strictly increasing and continuously differentiable in  $u_i$  and quasi-concave with respect to the policy variables, and  $p_i^k$  is the amount of good  $z$  required to produce one unit of transport infrastructure in country  $i$  (or, unit cost of infrastructure). The second constraint is the budget constraint of the planner.

The first order conditions with respect to the level of infrastructure and the price of infrastructure use are

$$-w_i t_i' (l_1 x_1^2 + l_2 x_2^1) = p_i^k \quad (2)$$

$$f_i = 0 \quad (3)$$

The LHS of (2) is the product of the quantity of goods 1 and 2 transported between the two countries and the marginal change in the transport cost by an investment in transport infrastructure in country  $i$ . The LHS is the marginal benefit of the investment, while the RHS is the marginal cost.

Condition (3) means that the price of infrastructure use should be zero. This is natural since marginal cost of usage, such as operation cost or congestion, does not exist in our model.

### 3.2 Free access, investment by national government (Regime F)

This regime has been discussed by Mun and Nakagawa (2007). In this regime, the government of country  $i$  maximizes the welfare of its citizens subject to the budget constraint while its price of infrastructure use is fixed to zero, i.e. the infrastructure is free access. The government chooses the level of infrastructure and collects a poll tax to finance the expenditure. We call this regime as the free access regime or regime F. The problem of the government is given by

$$\max_{k_i} v_i \left( p_i^i, p_j^j + w_i t_i + w_j t_j, w_i - \tau_i \right),$$

where

$$\tau_i = \frac{p_i^k k_i}{l_i}. \quad (4)$$

The first order condition of the above problem yields the following investment rule

$$-w_i t_i^j l_i x_i^j = p_i^k. \quad (5)$$

The LHS of equation (5) is the quantity of good  $j$  that country  $i$  imported from country  $j$  multiplied by marginal change in transport cost. Namely, the LHS is the marginal benefit to country  $i$  of an investment in the transport infrastructure. The RHS is the marginal cost of the investment. Comparison between conditions (2) and (5) reveals that investment rule under free access is inefficient since country  $i$  does not take account of the marginal benefit to country  $j$ .

Country  $i$ 's optimum level of investment is given as the solution to equation (5). Since the quantity of import goods depends on the transport cost in country  $j$ , the solution to equations (5) is written as

$$k_i = K_i^F(k_j, l_i w_i, p_i^k) \text{ for } i, j = 1, 2, j \neq i, \quad (6)$$

which is considered as country  $i$ 's reaction function. As shown in Mun and Nakagawa (2007), the investment on transport infrastructure is strategic complement, namely,

$$\frac{\partial K_i^F}{\partial k_j} > 0 \text{ for } i, j = 1, 2, j \neq i.$$

### 3.3 Pricing and investment by national government (Regime P)

In this regime, the government in country  $i$  chooses not only the level of transport infrastructure but also the price of infrastructure use. We call this regime as the pricing regime or regime P. The government maximizes the welfare of its citizens subject to the budget constraint. The problem of the government is given by

$$\max_{k_i, f_i} v_i(p_i^i, p_j^j + c_j + f_i + w_i t_i, w_i - \tau_i)$$

where  $\tau_i$  is obtained from the budget constraint, as follows

$$\tau_i = \frac{p_i^k k_i - f_i (l_i x_i^j + l_j x_j^i)}{l_i}. \quad (7)$$

The first order condition with respect to the price  $f_i$  yields

$$l_i x_i^j = -l_i \frac{\partial \tau_i}{\partial f_i}. \quad (8)$$

The LHS of (8) is the quantity of good  $j$  imported by country  $i$  that is equal to the loss in consumer surplus. A rise in the price of infrastructure use increases the price of the imported good in country  $i$ , which harms the welfare of the consumers. The RHS of (8) is the reduction in the poll tax caused by increase in revenue from infrastructure. Differentiating equation (7) with respect to  $\tau_i$  and  $f_i$  and rearranging the resulting equation yields the effect of a rise in the price of infrastructure use on the poll tax as

$$\frac{\partial \tau_i}{\partial f_i} = -\frac{l_i x_i^j + l_j x_j^i + f_i (l_i x_{ic}^j + l_j x_{jc}^i)}{l_i (1 - f_i x_{iw}^j)}, \quad (9)$$

where  $x_{ic}^j = \partial x_i^j / \partial (c_i + c_j)$ ,  $x_{jc}^i = \partial x_j^i / \partial (c_i + c_j)$ ,  $x_{iw}^j = \partial x_i^j / \partial w_i^g$ . The numerator of the RHS of (9) is the marginal revenue for the government. When there are no income effects on the demands for goods 1 and 2, the denominator becomes the size of the population. Then the reduction in the poll tax is equal to the per-capita increase in the government revenue. When there are income effects, a reduction in the poll tax raises the demand for goods 1 and 2, which further increases the infrastructure use and the government revenue. Thus, the reduction in the poll tax is greater than it is in the absence of income effects. Substituting equation (9) into (8), we have the pricing rule as

$$l_i x_i^j = \frac{l_i x_i^j + l_j x_j^i + f_i (l_i x_{ic}^j + l_j x_{jc}^i)}{1 - f_i x_{iw}^j}. \quad (10)$$

We now turn to the investment rule. From the first order condition, we get

$$-w_i t'_i l_i x_i^j = l_i \frac{\partial \tau_i}{\partial k_i}. \quad (11)$$

The LHS of (11) is the increase in country  $i$ 's consumer surplus from one unit increase in the capacity of transport infrastructure, which is the marginal benefit of the investment. The RHS is the marginal cost of investment. Condition (11) is the cost-benefit rule for the government that is concerned only with the welfare of its citizens.

The per-capita marginal cost of investment is given by differentiating equation (7) with respect to  $\tau_i$  and  $k_i$ :

$$\frac{\partial \tau_i}{\partial k_i} = \frac{p_i^k - f_i (l_i x_{ic}^j + l_j x_{jc}^i) w_i t'_i}{l_i (1 - f_i x_{iw}^j)}. \quad (12)$$

The numerator of the RHS of (12) is the net effect on the government expenditure of an one-unit investment on transport infrastructure. Note that the investment increases the trade volume and the revenue from the infrastructure. The second term of the numerator is the rise in revenue from infrastructure, which reduces the tax burden. Substituting (10) and (12) into (11) and rearranging the resulting equation, we have the investment rule as follows.

$$-w_i t'_i (l_i x_i^j + l_j x_j^i) = p_i^k. \quad (13)$$

This equation is identical to the first-best investment rule. The government that is concerned only about the welfare of its citizens takes into account of the benefit for other country's citizen in the end. This is because the investment generates additional benefit as described earlier: increase the revenue induced by expansion of trade volume. This revenue effect turns out to be equal to the benefit for other country's citizens.

Country  $i$ 's optimal level of investment and its optimal price of infrastructure use are the solutions to equations (10) and (13). These equations include the quantity of export good  $i$ , which depends on the disposable income and transport cost between two countries. Thus, the solutions are given by

$$k_i^P = K_i^P (c_j, w_j - \tau_j, l_i w_i, p_i^k), \quad (14)$$

$$f_i^P = F_i^P (c_j, w_j - \tau_j, l_i w_i, p_i^k) \text{ for } i, j = 1, 2, j \neq i, \quad (15)$$

where the super script  $P$  represents this regime.

The response of these functions to a change in the transport cost in country  $j$  is not straightforward. When the demand for import good does not depend on the disposable income ( $x_{iw}^j = 0$  for  $i, j = 1, 2, j \neq i$ ), the response is given in the following lemma. The proof of this lemma is shown in the Appendix A.

**Lemma 1** *Suppose that in each country  $i$ , the demand of import good  $j$  are independent of disposable income. Then, country  $i$ 's investment decisions in response to other countries policies are*

$$\frac{\partial K_i^P}{\partial k_j} > 0 \text{ and } \frac{\partial K_i^P}{\partial f_j} < 0$$

*Country  $i$ 's pricing decision depends on the shape of demand function for import goods.*

$$\frac{\partial F_i^P}{\partial k_j} \leq 0 \text{ and } \frac{\partial F_i^P}{\partial f_j} \geq 0 \iff l_j x_{jc}^i + f_i (l_i x_{icc}^j + l_j x_{jcc}^i) \geq 0,$$

where  $x_{icc}^j = \partial^2 x_{ic}^j / \partial (c_i + c_j)^2$  and  $x_{jcc}^i = \partial^2 x_{jc}^i / \partial (c_i + c_j)^2$ .

This lemma implies that when the demand for import good is linear, the RHS of the condition becomes negative. Then, country  $i$  increases its price of infrastructure use in response to a decrease in country  $j$ 's price, that is, the pricing decisions are strategic substitutes. When the demand is nonlinear and sufficiently convex, the pricing decisions may be strategic complements.

### 3.4 Private operation of transport infrastructure (Regime M)

Suppose a competitive auction in which the right to build and operate the transport infrastructure is awarded to the firm offering the highest bid. We assume that this auction is competitive in that there are sufficient number of equally productive firms, that no firm has a market power in the bidding process, and that there is no collusion between firms. Thus, the profit of the winner will be zero. We call this regime as the private operation regime or regime M. In this case, the winning firm should solve the following problem:

$$\max_{k_1, k_2, f} f (l_1 x_1^2 + l_2 x_2^1) - p_1^k k_1 - p_2^k k_2$$

The first order conditions are given by

$$f (l_i x_{ic}^j + l_j x_{jc}^i) w_i t'_i - p_i^k = 0, \quad (16)$$

$$l_1 x_1^2 + l_2 x_2^1 + f (l_1 x_{1c}^2 + l_2 x_{2c}^1) = 0. \quad (17)$$

From equations (16) and (17), we have the investment rule as

$$-w_i t'_i (l_i x_{ic}^j + l_j x_{jc}^i) = p_i^k \quad (18)$$

This investment rule is identical to the first-best rule as in the case of pricing and investment by national government. The LHS of (17) is the marginal revenue for the firm. Since there is no operation cost, any change in traffic does not affects the cost of the firm. Thus,



(17) is the condition of revenue maximization. It should be noted that when the demands for import goods do not depend on income, the pricing rule in this regime is identical to that in the pricing regime (regime P).<sup>2</sup>

Thus, we have the following lemma:

**Lemma 2** *Suppose that in each country  $i$ , the demand of import good  $j$  are independent of disposable income. Then, private operation and pricing and investment by national government yield the same outcome.*

Private operation by a single firm ignores the effect on consumers' welfare in the pricing decision, but avoids double margins in the independent pricing by governments. Lemma 2 shows that these positive and negative effects of the private operation are exactly offset in the absence of income effect.

### 3.5 User cost minimization (Regime U)

In this regime, a competitive bidding is designed so that the right to construct and operate the transport infrastructure is awarded to the firm proposing the plan that minimizes the user cost.<sup>3</sup> We call this regime as the user cost minimization regime or regime U. The bidder's problem is

$$\min_{f, k_1, k_2} f + w_1 t_1 + w_2 t_2$$

subject to

$$f (l_1 x_1^2 + l_2 x_2^1) - p_1^k k_1 - p_2^k k_2 = 0$$

The investment rule is again the same as in the first-best optimum.

$$-w_i t'_i (l_i x_i^j + l_j x_j^i) = p_i^k. \quad (19)$$

To interpret this result, (19) is rewritten as follows

$$\frac{-w_i t'_i}{p_i^k} = \frac{1}{l_i x_i^j + l_j x_j^i}. \quad (20)$$

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<sup>2</sup>When there are no income effects on the demands for import goods, the pricing rule in regime P is written as

$$l_j x_j^i + f_i (l_i x_{ic}^j + l_j x_{jc}^i) = 0 \text{ for } i, j = 1, 2, j \neq i.$$

Summing up the above equations for two countries, we have

$$l_1 x_1^2 + l_2 x_2^1 + (f_1 + f_2) (l_1 x_{1c}^2 + l_2 x_{1c}^1) = 0$$

Since  $f = f_1 + f_2$ , the above equation coincides with (17).

<sup>3</sup>Note that trade volume must be maximized by user cost minimization. Thus this regime is equivalent to the patronage maximization discussed in Verhoef (2007).

The above condition states that marginal benefit-cost ratios should be equalized across policy instruments to minimize the user cost: investment in infrastructure and price reduction. The LHS of the above equation is the marginal benefit-cost ratio of investment in country  $i$ : the benefit and cost are appeared respectively in the numerator and denominator. When the firm invests one unit on the infrastructure, the transport time decreases  $-w_i t'_i$  units. This investment incurs the cost of infrastructure,  $p_i^k$ . The RHS is the marginal benefit-cost ratio of price reduction. When the firm reduces the price by one unit, the transport costs reduces by one unit. This price reduction reduces the firm's revenue by the quantity of the infrastructure use that is considered as cost.

Substituting this investment rule in the zero profit condition yields the pricing rule:

$$f = -w_1 t'_1 k_1 - w_2 t'_2 k_2.$$

## 4 Evaluation of alternative regimes

To obtain the explicit result, we specify the form of the utility function as

$$u_i(x_i^i, x_i^j, z_i) = z_i - \frac{x_i^i}{\alpha_{ea}} \left[ \ln \left( \frac{x_i^i}{\alpha_{eb}} \right) - 1 \right] - \frac{x_i^j}{\alpha_{ma}} \left[ \ln \left( \frac{x_i^j}{\alpha_{mb}} \right) - 1 \right] \text{ for } i, j = 1, 2, j \neq i, \quad (21)$$

where  $\alpha_{ea}$ ,  $\alpha_{eb}$ ,  $\alpha_{ma}$ , and  $\alpha_{mb}$  are parameters representing the preferences for consumptions of the export and import goods. Then, the demand of a household in country  $i$  for import good  $j$  is given by

$$x_i^j = \alpha_{mb} \exp(-\alpha_{ma} p_i^j).$$

We also specify the function describing the transport technology as

$$t_i(k_i) = -\beta \ln \frac{k_i}{\bar{k}}, \quad (22)$$

$\bar{k}$  is the upper limit of the capacity of transport infrastructure ( $\bar{k} = 1$ ).

### 4.1 Analytical results

In this subsection, we present some analytical results derived for the specific forms of utility and transport technology functions.

#### 4.1.1 A symmetric economy

Suppose that the sizes and technologies of countries are identical. We present the ranking of the regimes for several criteria such as capacity of infrastructure, transport cost, and welfare. Since our specific utility function does not have an income effect, the private operation regime yields the same allocation as the pricing regime. Thus, we omit the results for the private operation regime. In the following, super scripts indicate the regimes: O represents the first best optimum, F the free access regime, P the pricing regime, and U the user cost minimization regime. The proofs of the following lemmas and proposition are given in Appendix B.

First, the levels of investment in the regimes are ranked as follows.

**Lemma 3** *In a symmetric economy, the levels of investment in the regimes satisfy the following relations*

$$\begin{aligned} k_i^O > k_i^U > k_i^F > k_i^P & \text{ if } \alpha_{ma}\beta w_i < \frac{1}{2} \log 2 \cong 0.346574, \\ k_i^O > k_i^U = k_i^F > k_i^P & \text{ if } \alpha_{ma}\beta w_i = \frac{1}{2} \log 2, & \text{ for } i = 1, 2, \\ k_i^O > k_i^F > k_i^U > k_i^P & \text{ if } \alpha_{ma}\beta w_i > \frac{1}{2} \log 2. \end{aligned}$$

The investment on transport infrastructure is lowest in the pricing regime. Although the investment rule in the pricing regime is identical with the first best rule, the investment level in the pricing regime is lower than that in the free access regime in which investment decision is not efficient. Note that the marginal benefit of investment (LHS of the expression for the investment rule) is proportional to the volume of trade. In the pricing regime, the volume of trade is smaller due to excessively high price of infrastructure use. This effect reduces the marginal benefit, and leads to smaller level of investment.

The ranking of the free access regime and the user cost minimization regime depends on the wage rate and the slope of demand function for import good and transport technology. If the demand is more elastic (smaller  $\alpha_{ma}$ ), transport technology is backward (smaller  $\beta$ ), or wage rate is lower (smaller  $w_i$ ), the level of investment in the user cost minimization is higher than that in the free access regime.

The transport cost depends not only on the level of investment but also on the price of infrastructure use. Thus, the ranking of transport cost is different from that of the capacity:

**Lemma 4** *In a symmetric economy, the transport costs in the regimes satisfy the following relations*

$$c^P > c^U > c^F > c^O,$$

where  $c^r = c_1^r + c_2^r$  for  $r = O, F, P,$  and  $U$ .

Unlike the levels of investment, the ranking is not contingent. The transport cost is highest under the pricing regime. User cost in the regime U is higher than that in the regime F. This result holds, even in the case that the level of investment is higher in the regime U. The price effect dominates the effect of larger capacity.

The utility function specified as (21) is quasi-linear form, thereby the utility is measured in monetary terms. So we define global welfare as the sum of utilities of all households in the economy, that is,  $W = l_1 v_1 + l_2 v_2$ . We have the ordering of the global welfare as follows:

**Proposition 1** *In a symmetric economy, the levels of the global welfare in the regimes are ranked as follows*

$$\begin{aligned} W^O > W^U > W^F > W^P & \text{ if } \alpha_{ma}\beta w_i < \theta \cong 0.229966, \\ W^O > W^U = W^F > W^P & \text{ if } \alpha_{ma}\beta w_i = \theta, & \text{ for } i = 1, 2, \\ W^O > W^F > W^U > W^P & \text{ if } \alpha_{ma}\beta w_i > \theta, \end{aligned}$$

where  $\theta$  is the solution of the following equation

$$\left(\frac{2}{e}\right)^{\frac{2\theta}{1-2\theta}} - 1 + \theta = 0.$$

Proposition 1 shows that the pricing by national governments (regime P) is less efficient than the free-access (regime F). This result is easily anticipated from Lemmas 3 and 4. Ohsawa (2000) obtains contrasting result that the pricing attains the higher welfare than the free-access. His model is different from ours in that the pricing and investment decisions are determined by voting. The second-best policy may be different depending on the parameters. When the wage rate, the effect of investment, and the demand for import good are low, the second-best regime in a symmetric economy is the user cost minimization regime. Otherwise, the second-best regime is the free access regime. However, the calibration for numerical analysis suggests that the value of  $(\alpha_{ma}\beta w_i)$  is very small (much smaller than 0.1). Therefore,  $W^U > W^F$  is more likely.

#### 4.1.2 An asymmetric economy

This paper examines the asymmetry between two countries in population size and the wage rates. We have some analytical results concerning the effect of difference in population size. Let us focus on the population distribution holding the sum of population of two countries constant. Let  $s_i$  be the country  $i$ 's share of population such that  $s_i = l_i / (l_1 + l_2)$ .

**Lemma 5** *Under the free-access regime, the effect of population share of country 1 on investment levels of two countries are*

$$\begin{aligned} \frac{\partial k_1^F}{\partial s_1} \begin{matrix} \geq \\ < \end{matrix} 0 &\iff s_1 \begin{matrix} \leq \\ \geq \end{matrix} 1 - \alpha_{ma}\beta w_2, \\ \frac{\partial k_2^F}{\partial s_1} \begin{matrix} \geq \\ < \end{matrix} 0 &\iff s_1 \begin{matrix} \leq \\ \geq \end{matrix} \alpha_{ma}\beta w_1, \end{aligned}$$

The effect on the user cost is

$$\frac{\partial c^F}{\partial s_1} \begin{matrix} \leq \\ \geq \end{matrix} 0 \iff s_1 \begin{matrix} \leq \\ \geq \end{matrix} \frac{w_1}{w_1 + w_2}.$$

In addition, suppose that the wage rate and the FOB price of export good in each country is identical, that is,  $w_1 = w_2$  and  $p_1^1 = p_2^2$ . Then, the global welfare in the free access regime satisfy

$$\frac{\partial W^F}{\partial s_1} \begin{matrix} \geq \\ < \end{matrix} 0 \iff s_1 \begin{matrix} \leq \\ \geq \end{matrix} \frac{1}{2}.$$

The effect of population distribution on the level of investment may be either positive or negative. It is likely that  $\partial k_1^F / \partial s_1 > 0$  and  $\partial k_2^F / \partial s_1 < 0$  unless population distribution is extremely uneven. In words, the level of investment is increasing with the population share of home country. The transport cost,  $c^F$ , is increasing with asymmetry. This is because investment in infrastructure is decreasing returns: transport cost reduction in the larger country is smaller than transport cost increase in the smaller country. Consequently the global welfare is decreased by increasing the asymmetry.

We turn to the other regimes.

**Lemma 6** *The effects of population distribution in regimes O, P, and U are*

$$\begin{aligned} \frac{\partial f_1^P}{\partial s_1} < 0, \frac{\partial f_2^P}{\partial s_1} > 0, \frac{\partial (f_1^P + f_2^P)}{\partial s_1} = 0, \\ \frac{\partial f^U}{\partial s_1} = 0, \\ \frac{\partial k_i^r}{\partial s_1} \begin{matrix} \geq \\ \leq \end{matrix} 0, \text{ and } \frac{\partial c^r}{\partial s_1} \begin{matrix} \leq \\ \geq \end{matrix} 0 \iff p_1^1 \begin{matrix} \geq \\ \leq \end{matrix} p_2^2, \quad i = 1, 2, r = O, P, U. \end{aligned}$$

In addition, when  $w_1 = w_2$  and  $p_1^1 = p_2^2$ , we have

$$\frac{\partial W^O}{\partial s_1} = \frac{\partial W^P}{\partial s_1} = \frac{\partial W^U}{\partial s_1} = 0.$$

Lemma 6 shows that if asymmetry between two countries is only in population size, then the investment levels, transport cost between two countries, and global welfare levels are independent from population distribution. This implies that the results of Lemmas 3, 4, and Proposition 1 concerning ranking between regimes O, P, U hold if the population distribution is asymmetric.

The effects of population distribution on the investment levels and transport cost depend on the difference in prices. This is explained as follows. In this model, the transport costs of goods 1 and 2 are identical. The ratio of the transported quantity of good 1 to that of good 2 is determined by the ratio of FOB price. Suppose that the FOB price of good 2 is lower than that of good 1. Then, a household in country 1 consumes more import good 2 than a household in country 2 consumes import good 1. An increase in the size of country 1 and corresponding decrease in the size of country 2 raises the quantity transported between the two countries. This increase in the transportation raises the investment and reduces the transport cost.

## 4.2 Numerical results

In this subsection, we evaluate the regimes in more general setting by means of numerical simulations. In the simulation, the parameters are set so as to describe the real world pictures. We choose the several observable indicators that are endogenous variables of the model, and set the parameter values such that the output of the model fits the prespecified (and reasonable) values of the indicators. Details are described in Appendix D. Table 1 represents the values of the parameters and indicators.

### 4.2.1 Asymmetry in country size

Figure 1 shows the results of simulations for various population distribution. Note that  $\alpha_{ma}\beta w_i < \frac{1}{2} \log 2 < \theta$  under the parameter values in Table 1. The horizontal axis measures country 1's share of the global population, that is,  $s_1 = l_1 / (l_1 + l_2)$ . Vertical dotted line is drawn so that it crosses the horizontal axis at  $s_1 = 0.5$  in order to verify the results

Table 1: Parameter values

$a_1^z, a_2^z$	1	$\alpha_{ma}$	0.833333
$a_1^1, a_2^2$	1	$\alpha_{mb}$	0.453047
$l_1 + l_2$	1	$\alpha_{ea}$	0.833333
$p_1^k$	0.026472	$\alpha_{eb}$	0.67957
$p_2^k$	0.026472	$\beta$	0.06

in lemmas and proposition for the symmetric case. Curve O represents the first best optimum, curve F the free access regime, curve P the pricing regime, and curve U the user cost minimization regime.

Figure 1(a) shows the capacity of transport infrastructure in country 1. In the free access regime, the investment of a country increases with its size, but the sum of investments of the two countries is almost the same as that in the symmetric economy. When country 1's population share is small, its investment in the free access regime is smaller than that in the pricing regime: Lemma 3 does not hold when population distribution is quite uneven.

Figure 1(b) illustrates the transport time. When the global population distributes uniformly, the transport time in the free access regime is lower than that in the pricing regime. As the distribution becomes more uneven, this relation is reversed.

Figure 1(c) depicts the transport cost. In most situation, the transport cost in the pricing regime is significantly higher than those in the other regimes due to high price for infrastructure use. For example, in the symmetric case, the price in the pricing regime is 1.2, while the price in the user cost minimization regime is 0.12. In an asymmetric economy, the transport time and cost in the free access regime are higher than those in the symmetric economy. The reason is that the transport technology is decreasing return to scale: the more a country invests, the less the effect of investment is. When the population distribution is extremely uneven, the transport cost in the free access regime exceeds that in the pricing regime.

Figure 1(d) shows the efficiency ratios of the four regimes. The efficiency ratio measured on the vertical axis is calculated by the following formula:

$$\text{Efficiency ratio} = \frac{W^r - W^P}{W^O - W^P} \text{ for } r = F, U.$$

This indicator is utilized for evaluating the relative efficiency of regimes. Note that global welfare in the pricing regime turns out lowest in almost all cases. The efficiency ratio indicates welfare gain from this level relative to the maximum gain. This figure suggests that welfare loss in the pricing regime is very large. This is due to the distortion caused by pricing behavior, as explained for Figure 1(c).

In this setting, the user cost minimization regime is the second best. Overall, numerical results suggest that the ranking of regimes is not affected by introducing asymmetry in population size. However the countries might not have the incentive to adopt this regime. Figure 1(e) shows the welfare of the representative household in country 1. When the population in country 1 is sufficiently small, the household in country 1 most prefers the pricing regime.

### 4.2.2 Asymmetry in productivity

Let us examine the effect of the asymmetry in productivity, which is represented by the difference in wage rates between countries. Consider an economy which differs from the symmetric economy only in the wage rate in country 2. Other parameters are identical with those in Table 1. Figure 2 illustrates how the relative wage affects the investment, transport, and welfare. The horizontal axis of this figure is the relative wage rate of country 2. The wage rate of country 1 is fixed at one. The notation of the curves is the same as those in Figure 1.

Figure 2(a) depicts the investment in country 1. In each regime, country 1 reduces its investment as the wage rate in country 2 rises. Figure 2(b) shows the investment in country 2. In each regime, country 2's investment increases with its wage rate. Namely, a country increases its investment in response to an increase in home country's wage, and decreases in response to an increase in the other country's wage. In addition, Figure 2(b) indicates that country 2 invests more in the pricing regime than in the free access regime when its wage rate is low. In other cases, the level of investment in the pricing regime is lower. Figure 2(c) shows that ranking of transport time among regimes is not affected by the difference in wage rates. It is seen that transport time in each regime is decreasing with wage rate in country 2, but its level is relatively stable against changes in wage differential. This is because increase in investment of one country is offset by decrease in other country, as shown in Figure 2 (a) and (b). Figure 2(d) depicts the transport cost. In contrast to the transport time, the transport cost increases as country 2's wage rate raises. Figure 2(e) illustrates the efficiency ratio. For any wage rate of country 2, the ratio in the user cost minimization regime is higher than that in the free access regime. As the wage rate of country 2 falls, the values of ratios for both regimes become close to one.

## 5 Conclusion

If the use of infrastructure is free of charge, the national government does not take into account the benefit of investment in other country, which leads to under-investment in terms of resource allocation for two-country economy as a whole. On the other hand, it is shown that the investment rule become efficient if the government levies charge for infrastructure use. The problem with charging is that the price would be inefficiently high, which negatively affects the level of investment despite the efficient rule. Comparison of these two regimes, free access vs. pricing by government, shows that the distortion of pricing in the latter exceeds the loss owing to lack of incentive in investment decision in the former regime. However, it is also shown that user cost minimization regime attains higher welfare than free access. This result suggests that more efficient mechanism with pricing could be designed. We need to make further efforts in the future to explore the possibility of finding better mechanisms.

## Appendix A

In this appendix, we give the proof of Lemma 1. For conciseness, define the quantity of good  $j$  imported by country  $i$  as  $N_i = l_i x_i^j$ . We also let the sum of  $N_1$  and  $N_2$  denoted by

$N$ . We assume that the demands for import good do not depend on the disposable income. The first order conditions are written as follows

$$\frac{\partial v_i}{\partial f_i} = \frac{v_{iw}}{l_i} (N_j + f_i N_c) = 0, \quad (23)$$

$$\frac{\partial v_i}{\partial k_i} = \frac{v_{iw}}{l_i} (-N_i w_i t'_i - p_i^k + f_i N_c w_i t'_i) = 0, \quad (24)$$

where  $N_c = N_{ic} + N_{jc}$ ,  $N_{ic} = \partial N_i / \partial (c_1 + c_2)$ ,  $N_{jc} = \partial N_j / \partial (c_1 + c_2)$ . Differentiating (23) and (24) with respect to  $f_i$ ,  $k_i$ , and  $c_j$  yields

$$\frac{v_{iw}}{l_i} \{(N_{ic} + 2N_{jc} + f_i N_{cc}) df_i + (N_{jc} + f_i N_{cc}) w_i t'_i dk_i + (N_{jc} + f_i N_{cc}) dc_j\} = 0, \quad (25)$$

and

$$\begin{aligned} \frac{v_{iw}}{l_i} \{(N_{jc} + f_i N_{cc}) w_i t'_i df_i + [- (N_{ic} + f_i N_{cc}) (w_i t'_i)^2 - N w_i t''_i] dk_i \\ + (-N_{ic} + f_i N_{cc}) w_i t'_i dc_j\} = 0 \end{aligned} \quad (26)$$

where  $N_{cc} = N_{icc} + N_{jcc}$ ,  $N_{icc} = \partial^2 N_i / \partial (c_1 + c_2)^2$ ,  $N_{jcc} = \partial^2 N_j / \partial (c_1 + c_2)^2$ . Solving equations (25) and (26) with respect to  $df_i$  and  $dk_i$ , we get

$$\begin{aligned} \frac{\partial F_i^P}{\partial c_j} &= \frac{(N_{jc} + f_i N_{cc}) N w_i t''_i}{\Delta}, \\ \frac{\partial K_i^P}{\partial c_j} &= \frac{(N_c)^2 w_i t'_i}{\Delta}, \end{aligned}$$

where

$$\Delta = \begin{vmatrix} N_{ic} + 2N_{jc} + f_i N_{cc} & (N_{jc} + f_i N_{cc}) w_i t'_i \\ (N_{jc} + f_i N_{cc}) w_i t'_i & - (N_{ic} + f_i N_{cc}) (w_i t'_i)^2 - N w_i t''_i \end{vmatrix} > 0.$$

The fact that  $\Delta$  is positive follows from the second order condition of the national welfare maximization problem. Thus, we get

$$\begin{aligned} \frac{\partial F_i^P}{\partial c_j} &\geq 0 \text{ if } N_{jc} + f_i N_{cc} = l_j x_{jc}^i + f_i (l_i x_{icc}^j + l_j x_{jcc}^i) \geq 0, \\ \frac{\partial K_i^P}{\partial c_j} &< 0. \end{aligned}$$

Since  $w_i t'_i$  is negative, the lemma follows from the above equations.

## Appendix B

In this appendix, we give the proofs of the lemmas and proposition.



### Proof of Lemma 3

This lemma gives the ordering of the levels of investment on transport infrastructure in a symmetric economy. In this economy, we have

$$w_1 = w_2, l_1 = l_2, a_{11} = a_{22}.$$

From the stability condition of the equilibrium in the free access and the pricing regime, we have

$$0 < \alpha_{ma}\beta w_1 < \frac{1}{2}. \quad (27)$$

The levels of investment are given by

$$\begin{aligned} k_i^O &= \left( \frac{2l_1 e^{-\alpha_{ma} p_2^2} \alpha_{mb} \beta w_1}{p_1^k} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}}, \\ k_i^F &= \left( \frac{l_1 e^{-\alpha_{ma} p_2^2} \alpha_{mb} \beta w_1}{p_1^k} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}}, \\ k_i^P &= \left( \frac{2l_1 e^{-\alpha_{ma} p_2^2} \alpha_{mb} \beta w_1}{e p_1^k} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}}, \\ k_i^U &= \left( \frac{2l_1 e^{-\alpha_{ma} p_2^2} \alpha_{mb} \beta w_1}{e^{2\alpha_{ma}\beta w_1} p_1^k} \right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}}. \end{aligned}$$

Then, we have

$$\begin{aligned} \frac{k_i^O}{k_i^U} &= \exp\left(\frac{2\alpha_{ma}\beta w_1}{1-2\alpha_{ma}\beta w_1}\right) > 1, \\ \frac{k_i^U}{k_i^P} &= e > 1, \\ \frac{k_i^O}{k_i^F} &= 2^{\frac{1}{1-2\alpha_{ma}\beta w_1}} > 1, \\ \frac{k_i^F}{k_i^P} &= \left(\frac{e}{2}\right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}} > 1. \end{aligned}$$

Thus, we have

$$k_i^O > k_i^U > k_i^P \text{ and } k_i^O > k_i^U > k_i^P.$$

The ordering of the levels of the pricing regime and of the user cost minimization regime depends on  $\alpha_{ma}\beta w_1$ , namely,

$$\frac{k_i^F}{k_i^U} = \left(\frac{e^{2\alpha_{ma}\beta w_1}}{2}\right)^{\frac{1}{1-2\alpha_{ma}\beta w_1}} > 1 \text{ if and only if } \frac{e^{2\alpha_{ma}\beta w_1}}{2} > 1.$$

Thus, we have the lemma.

## Proof of Lemma 4

This lemma gives the ordering of the user costs in the regimes. The user costs are given by

$$\begin{aligned} c^O &= \frac{-2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \log \left( \frac{2l_1 \exp(-\alpha_{ma}p_2^2) \alpha_{mb}\beta w_1}{p_1^k} \right), \\ c^F &= \frac{-2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \log \left( \frac{l_1 \exp(-\alpha_{ma}p_2^2) \alpha_{mb}\beta w_1}{p_1^k} \right), \\ c^P &= \frac{2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \left[ \frac{1}{2\alpha_{ma}\beta w_1} - \log \left( \frac{2l_1 \exp(-\alpha_{ma}p_2^2) \alpha_{mb}\beta w_1}{p_1^k} \right) \right], \\ c^U &= \frac{2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \left[ 1 - \log \left( \frac{2l_1 \exp(-\alpha_{ma}p_2^2) \alpha_{mb}\beta w_1}{p_1^k} \right) \right]. \end{aligned}$$

Then, we have

$$\begin{aligned} c^O - c^F &= \frac{-2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} \log 2 < 0, \\ c^F - c^U &= \frac{-2\beta w_1}{1 - 2\alpha_{ma}\beta w_1} (1 - \log 2) < 0, \\ c^U - c^P &= \frac{-1}{\alpha_{ma}} < 0. \end{aligned}$$

Thus, we have

$$c^O < c^F < c^U < c^P.$$

## Proof of Proposition 1

Finally, we give the welfare ordering of the regimes. In a symmetric economy, the levels of global welfare in the regimes are given by

$$\begin{aligned} W^O &= \bar{W} + 2l_1 e^{-\alpha_{ma}p_2^2} \frac{\alpha_{mb}}{\alpha_{ma}} (1 - 2\alpha_{ma}\beta w_1) \left( \frac{2l_1 e^{-\alpha_{ma}p_2^2} \alpha_{mb}\beta w_1}{p_1^k} \right)^{\frac{2\alpha_{ma}\beta w_1}{1 - 2\alpha_{ma}\beta w_1}}, \\ W^F &= \bar{W} + 2l_1 e^{-\alpha_{ma}p_2^2} \frac{\alpha_{mb}}{\alpha_{ma}} (1 - \alpha_{ma}\beta w_1) \left( \frac{l_1 e^{-\alpha_{ma}p_2^2} \alpha_{mb}\beta w_1}{p_1^k} \right)^{\frac{2\alpha_{ma}\beta w_1}{1 - 2\alpha_{ma}\beta w_1}}, \\ W^P &= \bar{W} + \frac{4l_1 e^{-\alpha_{ma}p_2^2}}{e} \frac{\alpha_{mb}}{\alpha_{ma}} (1 - \alpha_{ma}\beta w_1) \left( \frac{2l_1 e^{-\alpha_{ma}p_2^2} \alpha_{mb}\beta w_1}{e p_1^k} \right)^{\frac{2\alpha_{ma}\beta w_1}{1 - 2\alpha_{ma}\beta w_1}}, \\ W^U &= \bar{W} + 2l_1 e^{-\alpha_{ma}p_2^2} \frac{\alpha_{mb}}{\alpha_{ma}} \left( \frac{2l_1 e^{-\alpha_{ma}p_2^2} \alpha_{mb}\beta w_1}{e p_1^k} \right)^{\frac{2\alpha_{ma}\beta w_1}{1 - 2\alpha_{ma}\beta w_1}}, \end{aligned}$$

where

$$\bar{W} = 2 \left( l_1 w_1 + l_1 \frac{\alpha_{eb}}{\alpha_{ea}} \exp(-\alpha_{ea}p_1^1) \right).$$

Then, we have

$$\frac{W^U - \bar{W}}{W^P - \bar{W}} = \frac{e}{2(1 - \alpha_{ma}\beta w_1)} > 1,$$

$$\frac{W^F - \bar{W}}{W^P - \bar{W}} = \left(\frac{e}{2}\right)^{\frac{1}{1 - 2\alpha_{ma}\beta w_1}} > 1.$$

Thus, we get  $W^U > W^P$  and  $W^F > W^P$ .

Comparing the first best global welfare with the global welfare in the user cost minimization regime yields

$$\frac{W^O - \bar{W}}{W^U - \bar{W}} = (1 - 2\alpha_{ma}\beta w_1) e^{\frac{2\alpha_{ma}\beta w_1}{2\alpha_{ma}\beta w_1 - 1}}.$$

Consider the logarithm of the RHS. The RHS is greater than one if the logarithm is positive. The logarithm is given by

$$\log(1 - 2\alpha_{ma}\beta w_1) - f(1 - 2\alpha_{ma}\beta w_1),$$

where

$$f(x) = \frac{x - 1}{x}.$$

Investigating the slope and the intercept of  $f(x)$ , we obtain  $\log x > f(x)$  for  $0 < x < 1$ .<sup>4</sup> This yields  $W^O > W^U$ .

Comparing the first best global welfare with the global welfare in the free access regime yields

$$\frac{W^F - \bar{W}}{W^O - \bar{W}} = \left(\frac{1 - \alpha_{ma}\beta w_1}{1 - 2\alpha_{ma}\beta w_1}\right) 2^{\frac{2\alpha_{ma}\beta w_1}{2\alpha_{ma}\beta w_1 - 1}}$$

$$= g(\alpha_{ma}\beta w_1),$$

where

$$g(x) = \left(\frac{1 - x}{1 - 2x}\right) 2^{\frac{2x}{2x - 1}}.$$

Then, we have

$$g(0) = 1 \text{ and } g'(x) < 0 \text{ for } 0 < x < 1.$$

This yields

$$g(x) < 1 \text{ for } 0 < x < 1.$$

Thus, we obtain  $W^O > W^F$ . To summarize the results, we get

$$W^O > W^F > W^P \text{ and } W^O > W^U > W^P.$$

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<sup>4</sup>We have  $\log 1 = f(1) = 0$  and

$$\frac{d}{dx} \log x - f'(x) = \frac{x - 1}{x^2} < 0 \text{ for } 0 < x < 1.$$

Thus, we have

$$\log x < f(x) \text{ for } 0 < x < 1.$$

Finally, let us consider the welfare ordering of the free access regime and the user cost minimization regime. We have

$$W^U - W^F = 2l_1 e^{-\alpha_{ma} p_2^2} \frac{\alpha_{mb}}{\alpha_{ma}} \left( \frac{l_1 e^{-\alpha_{ma} p_2^2} \alpha_{mb} \beta w_1}{p_1^k} \right)^{\frac{2\alpha_{ma} \beta w_1}{1-2\alpha_{ma} \beta w_1}} \left[ \left( \frac{2}{e} \right)^{\frac{2\alpha_{ma} \beta w_1}{1-2\alpha_{ma} \beta w_1}} - (1 - \alpha_{ma} \beta w_1) \right].$$

The bracket of the RHS is rewritten as

$$\left( \frac{2}{e} \right)^{\frac{2\alpha_{ma} \beta w_1}{1-2\alpha_{ma} \beta w_1}} - (1 - \alpha_{ma} \beta w_1) = h(\alpha_{ma} \beta w_1) - (1 - \alpha_{ma} \beta w_1),$$

where

$$h(x) = \left( \frac{2}{e} \right)^{\frac{2x}{1-2x}}.$$

The function  $h(x)$  satisfies

$$h(0) = 1, \lim_{x \rightarrow \frac{1}{2}} h(x) = 0,$$

$$h'(0) > -1,$$

$$h'(x) < 0 \text{ for } 0 < x < \frac{1}{2}.$$

Thus there is a unique point  $\theta \in (0, \frac{1}{2})$  such that

$$h(\theta) - (1 - \theta) = 0.$$

We also have

$$h(x) - (1 - x) > 0 \text{ for } x < \theta,$$

$$h(x) - (1 - x) < 0 \text{ for } x > \theta.$$

Thus, we have

$$W^U > W^F \text{ for } \alpha_{ma} \beta w_1 < \theta,$$

$$W^U = W^F \text{ for } \alpha_{ma} \beta w_1 = \theta,$$

$$W^U < W^F \text{ for } \alpha_{ma} \beta w_1 > \theta.$$

## Appendix C

In this appendix, we give the proof of Lemmas 5 and 6.

## 5.1 Proof of Lemma 5

In the free access regime, the levels of investment in countries 1 and 2 satisfy

$$k_1^F = \left[ \frac{s_1 (l_1 + l_2) \alpha_{mb} \beta w_1}{p_1^k e^{\alpha_{ma} p_2^2}} \right]^{\frac{1 - \alpha_{ma} \beta w_2}{1 - \alpha_{ma} \beta (w_1 + w_2)}} \left[ \frac{(1 - s_1) (l_1 + l_2) \alpha_{mb} \beta w_2}{p_2^k e^{\alpha_{ma} p_1^1}} \right]^{\frac{\alpha_{ma} \beta w_2}{1 - \alpha_{ma} \beta (w_1 + w_2)}},$$

$$k_2^F = \left[ \frac{s_1 (l_1 + l_2) \alpha_{mb} \beta w_1}{p_1^k e^{\alpha_{ma} p_2^2}} \right]^{\frac{\alpha_{ma} \beta w_1}{1 - \alpha_{ma} \beta (w_1 + w_2)}} \left[ \frac{(1 - s_1) (l_1 + l_2) \alpha_{mb} \beta w_2}{p_2^k e^{\alpha_{ma} p_1^1}} \right]^{\frac{1 - \alpha_{ma} \beta w_1}{1 - \alpha_{ma} \beta (w_1 + w_2)}}.$$

And the transport cost between two countries is given by

$$c^F = -\beta w_1 \ln k_1^F - \beta w_2 \ln k_2^F,$$

$$W^F = \bar{W}$$

$$+ \left( \frac{\alpha_{mb}}{\alpha_{ma}} \right) (l_1 + l_2) \left[ (1 - \alpha_{ma} \beta w_1) e^{-\alpha_{ma} p_2^2} s_1 + (1 - \alpha_{ma} \beta w_2) e^{-\alpha_{ma} p_1^1} (1 - s_1) \right]$$

$$\times \left[ \frac{s_1 (l_1 + l_2) \alpha_{mb} \beta w_1}{p_1^k e^{\alpha_{ma} p_2^2}} \right]^{\frac{\alpha_{ma} \beta w_1}{1 - \alpha_{ma} \beta (w_1 + w_2)}} \left[ \frac{(1 - s_1) (l_1 + l_2) \alpha_{mb} \beta w_2}{p_2^k e^{\alpha_{ma} p_1^1}} \right]^{\frac{\alpha_{ma} \beta w_2}{1 - \alpha_{ma} \beta (w_1 + w_2)}}.$$

Differentiating the above equations with respect to  $s_1$  yields

$$\frac{1}{k_1^F} \frac{\partial k_1^F}{\partial s_1} = \frac{1 - \alpha_{ma} \beta w_2 - s_1}{[1 - \alpha_{ma} \beta (w_1 + w_2)] s_1 (1 - s_1)},$$

$$\frac{1}{k_2^F} \frac{\partial k_2^F}{\partial s_1} = \frac{\alpha_{ma} \beta w_1 - s_1}{[1 - \alpha_{ma} \beta (w_1 + w_2)] s_1 (1 - s_1)},$$

$$\frac{\partial c^F}{\partial s_1} = \frac{\beta (w_1 + w_2)}{[1 - \alpha_{ma} \beta (w_1 + w_2)] s_1 (1 - s_1)} \left( s_1 - \frac{w_1}{w_1 + w_2} \right).$$

Thus, we have

$$\frac{\partial k_1^F}{\partial s_1} \geq 0 \iff s_1 \leq 1 - \alpha_{ma} \beta w_2,$$

$$\frac{\partial k_2^F}{\partial s_1} \geq 0 \iff s_1 \geq \alpha_{ma} \beta w_1,$$

$$\frac{\partial c^F}{\partial s_1} \leq 0 \iff s_1 \leq \frac{w_1}{w_1 + w_2},$$

which constitute the first half of lemma 5. The global welfare is given by

$$W^F = \bar{W}$$

$$+ \left( \frac{\alpha_{mb}}{\alpha_{ma}} \right) (l_1 + l_2) \left[ (1 - \alpha_{ma} \beta w_1) e^{-\alpha_{ma} p_2^2} s_1 + (1 - \alpha_{ma} \beta w_2) e^{-\alpha_{ma} p_1^1} (1 - s_1) \right]$$

$$\times \left[ \frac{s_1 (l_1 + l_2) \alpha_{mb} \beta w_1}{p_1^k e^{\alpha_{ma} p_2^2}} \right]^{\frac{\alpha_{ma} \beta w_1}{1 - \alpha_{ma} \beta (w_1 + w_2)}} \left[ \frac{(1 - s_1) (l_1 + l_2) \alpha_{mb} \beta w_2}{p_2^k e^{\alpha_{ma} p_1^1}} \right]^{\frac{\alpha_{ma} \beta w_2}{1 - \alpha_{ma} \beta (w_1 + w_2)}}.$$

If  $w_1 = w_2$  and  $p_1^1 = p_2^2$ , differentiation of the above equation with respect to  $s_1$  yields

$$\begin{aligned} \frac{\partial W^F}{\partial s_1} &= \left( \frac{\alpha_{mb}}{\alpha_{ma}} \right) \left[ \frac{s_1 (l_1 + l_2) \alpha_{mb} \beta w_1}{p_1^k e^{\alpha_{ma} p_1^1}} \right]^{\frac{\alpha_{ma} \beta w_1}{1 - 2\alpha_{ma} \beta w_1}} \left[ \frac{(1 - s_1) (l_1 + l_2) \alpha_{mb} \beta w_1}{p_2^k e^{\alpha_{ma} p_1^1}} \right]^{\frac{\alpha_{ma} \beta w_1}{1 - 2\alpha_{ma} \beta w_1}} \\ &\quad \times \frac{2(1 - \alpha_{ma} \beta w_1) e^{-\alpha_{ma} p_1^1} (l_1 + l_2) \alpha_{ma} \beta w_1}{(1 - 2\alpha_{ma} \beta w_1) s_1 (1 - s_1)} \left( \frac{1}{2} - s_1 \right), \end{aligned}$$

which is the latter half of lemma 5.

## Proof of Lemma 6

The prices of infrastructure use satisfy

$$\begin{aligned} f^O &= 0, \\ f_i^P &= \frac{l_j e^{-\alpha_{ma} p_i^1}}{l_1 e^{-\alpha_{ma} p_2^2} + l_2 e^{-\alpha_{ma} p_1^1}}, \quad f_1^P + f_2^P = \frac{1}{\alpha_{ma}}, \quad \text{for } i = 1, 2, \\ f^U &= \beta (w_1 + w_2). \end{aligned}$$

The levels of investment in the optimum, pricing, and user cost minimization regimes are given by

$$\begin{aligned} k_1^O &= \left[ \frac{(l_1 + l_2) \left( s_1 e^{-\alpha_{ma} p_2^2} + (1 - s_1) e^{-\alpha_{ma} p_1^1} \right) \alpha_{mb} \beta w_1}{p_1^k} \right]^{\frac{1 - \alpha_{ma} \beta w_2}{1 - \alpha_{ma} \beta (w_1 + w_2)}} \\ &\quad \times \left[ \frac{(l_1 + l_2) \left( s_1 e^{-\alpha_{ma} p_2^2} + (1 - s_1) e^{-\alpha_{ma} p_1^1} \right) \alpha_{mb} \beta w_2}{p_2^k} \right]^{\frac{\alpha_{ma} \beta w_2}{1 - \alpha_{ma} \beta (w_1 + w_2)}}, \\ k_2^O &= \left[ \frac{(l_1 + l_2) \left( s_1 e^{-\alpha_{ma} p_2^2} + (1 - s_1) e^{-\alpha_{ma} p_1^1} \right) \alpha_{mb} \beta w_1}{p_1^k} \right]^{\frac{\alpha_{ma} \beta w_1}{1 - \alpha_{ma} \beta (w_1 + w_2)}} \\ &\quad \times \left[ \frac{(l_1 + l_2) \left( s_1 e^{-\alpha_{ma} p_2^2} + (1 - s_1) e^{-\alpha_{ma} p_1^1} \right) \alpha_{mb} \beta w_2}{p_2^k} \right]^{\frac{1 - \alpha_{ma} \beta w_1}{1 - \alpha_{ma} \beta (w_1 + w_2)}}, \\ k_i^P &= \exp \left( \frac{-1}{1 - \alpha_{ma} \beta (w_1 + w_2)} \right) k_i^O, \\ k_i^U &= \exp \left( \frac{-\alpha_{ma} \beta (w_1 + w_2)}{1 - \alpha_{ma} \beta (w_1 + w_2)} \right) k_i^O, \quad \text{for } i = 1, 2. \end{aligned}$$

Differentiating the above equations with respect to  $s_1$  yields

$$\frac{1}{k_i^r} \frac{\partial k_i^r}{\partial s_1} = \frac{(l_1 + l_2) e^{-\alpha_{ma} p_1^1} \left[ e^{\alpha_{ma} (p_1^1 - p_2^2)} - 1 \right]}{[1 - \alpha_{ma} \beta (w_1 + w_2)] (l_1 e^{-\alpha_{ma} p_2^2} + l_2 e^{-\alpha_{ma} p_1^1})} \quad \text{for } i = 1, 2; r = O, P, U. \quad (28)$$

Similarly, differentiating the transport cost with respect to  $s_1$  yields

$$\frac{\partial c^r}{\partial s_1} = \frac{-\beta(w_1 + w_2)(l_1 + l_2)e^{-\alpha_{ma}p_1^1} \left[ e^{\alpha_{ma}(p_1^1 - p_2^2)} - 1 \right]}{[1 - \alpha_{ma}\beta(w_1 + w_2)](l_1 e^{-\alpha_{ma}p_2^2} + l_2 e^{-\alpha_{ma}p_1^1})}, \text{ for } r = O, P, U. \quad (29)$$

Since the denominators of the RHSs of the equations (28) and (25) are positive, the signs of  $\frac{1}{k_i^r} \frac{\partial k_i^r}{\partial s_1}$  and  $\frac{\partial c^r}{\partial s_1}$  depend on the signs of the brackets of the numerator,  $\left[ e^{\alpha_{ma}(p_1^1 - p_2^2)} - 1 \right]$ . Namely, we have

$$\frac{\partial k_i^r}{\partial s_1} \geq 0 \text{ and } \frac{\partial c^r}{\partial s_1} \leq 0 \iff e^{\alpha_{ma}(p_1^1 - p_2^2)} \geq 1 \text{ for } i = 1, 2, r = O, P, U.$$

When  $w_1 = w_2$  and  $p_1^1 = p_2^2$ , the global welfare satisfies

$$\begin{aligned} \bar{W} &= (l_1 + l_2) \left( w_1 + \frac{\alpha_{eb} e^{-\alpha_{ea} p_1^1}}{\alpha_{ea}} \right), \\ W^O &= \bar{W} \\ &+ \left( \frac{\alpha_{mb}}{\alpha_{ma}} \right) (1 - 2\alpha_{ma}\beta w_1) \left[ (l_1 + l_2) e^{-\alpha_{ma} p_1^1} \right]^{\frac{1}{1 - 2\alpha_{ma}\beta w_1}} \left[ \frac{(\alpha_{mb}\beta w_1)^2}{p_1^k p_2^k} \right]^{\frac{\alpha_{ma}\beta w_1}{1 - \alpha_{ma}\beta w_1}} \\ W^P &= \bar{W} \\ &+ \left( \frac{2\alpha_{mb}}{e\alpha_{ma}} \right) (1 - \alpha_{ma}\beta w_1) \left[ (l_1 + l_2) e^{-\alpha_{ma} p_1^1} \right]^{\frac{1}{1 - 2\alpha_{ma}\beta w_1}} \left[ \frac{(\alpha_{mb}\beta w_1)^2}{e^2 p_1^k p_2^k} \right]^{\frac{\alpha_{ma}\beta w_1}{1 - \alpha_{ma}\beta w_1}} \\ W^U &= \bar{W} \\ &+ \left( \frac{\alpha_{mb}}{\alpha_{ma}} \right) \left[ (l_1 + l_2) e^{-\alpha_{ma} p_1^1} \right]^{\frac{1}{1 - 2\alpha_{ma}\beta w_1}} \left[ \frac{(\alpha_{mb}\beta w_1)^2}{e^2 p_1^k p_2^k} \right]^{\frac{\alpha_{ma}\beta w_1}{1 - \alpha_{ma}\beta w_1}} \end{aligned}$$

Thus, we have

$$\frac{\partial W^r}{\partial s_1} = 0, \text{ for } r = O, P, U.$$

## Appendix D

The indicators are the openness of trade, the ratio of transport cost between two countries to the FOB price, the price elasticity of demand for import good, and the ratio of the government expenditure on transport infrastructure to the national income. The openness of country  $i$  is defined by the expenditure ratio of import good  $j$ . In this simulation, we set the openness of both countries at 0.2, the transport cost ratio at 0.2, the elasticity at 1, and the expenditure ratio at 0.01. The parameters are chosen such that the four indexes becomes these values when the infrastructure is free access cases in a symmetric economy with identical countries.

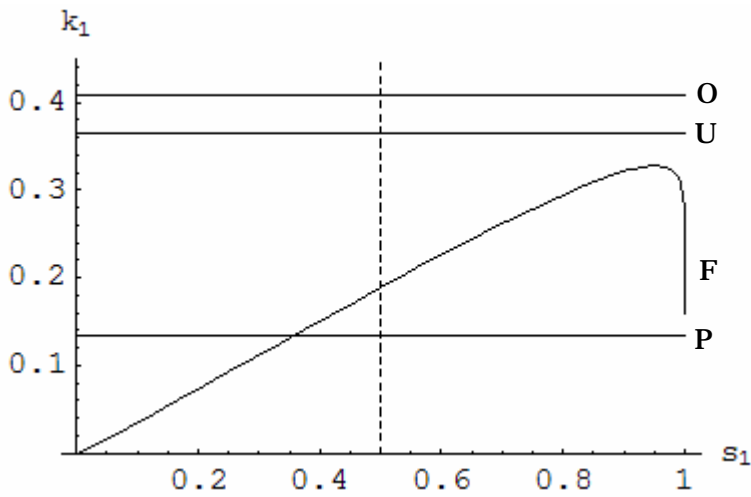
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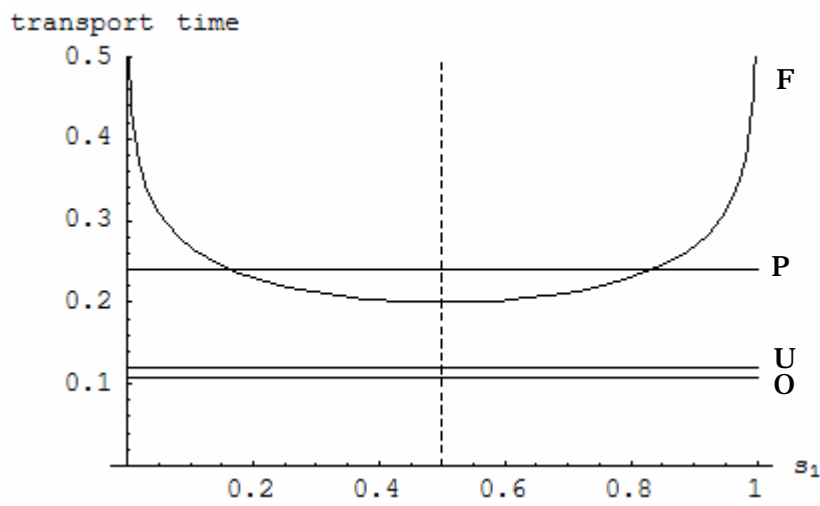
Figure 1 Effects of the population distribution

(a) Investment in country 1

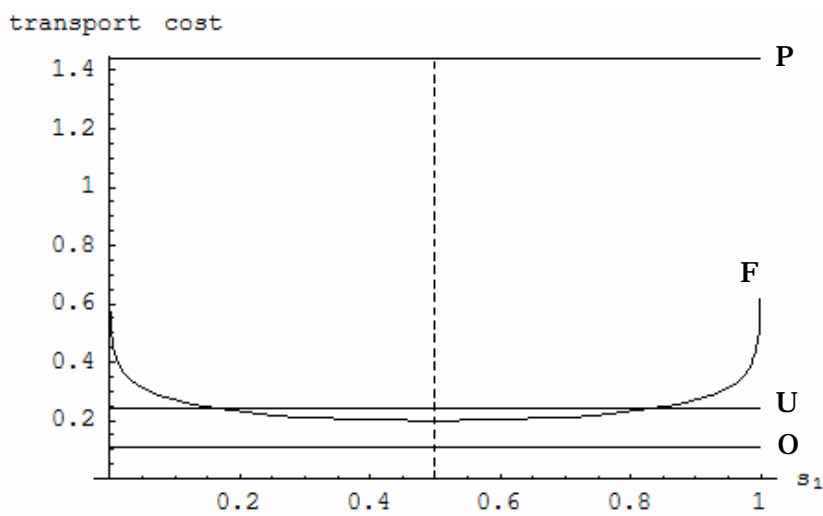


O: Optimum  
 F: Free access  
 P: Pricing and investment  
 by national government  
 U: User cost minimization

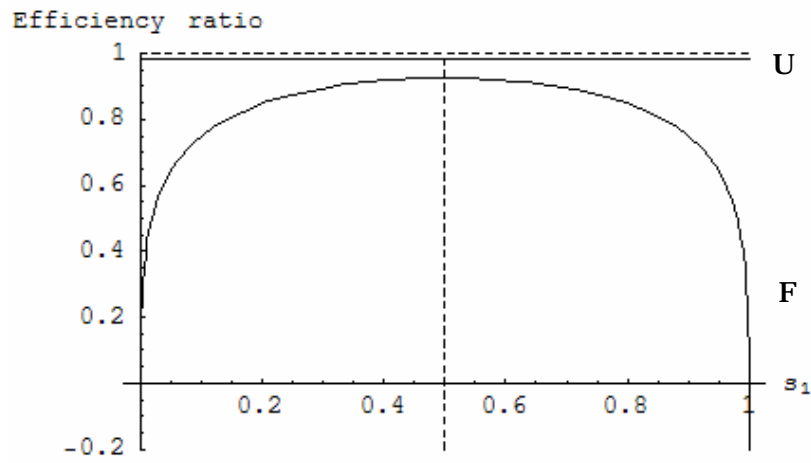
(b) Transport time



(c) Transport cost



(d) Efficiency ratio



O: Optimum  
F: Free access  
P: Pricing and investment  
by national government  
U: User cost minimization

(e) National welfare of country 1

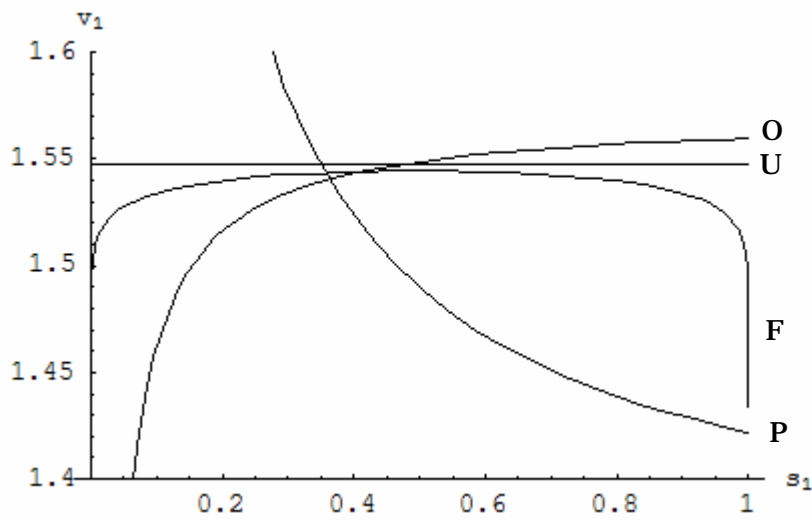
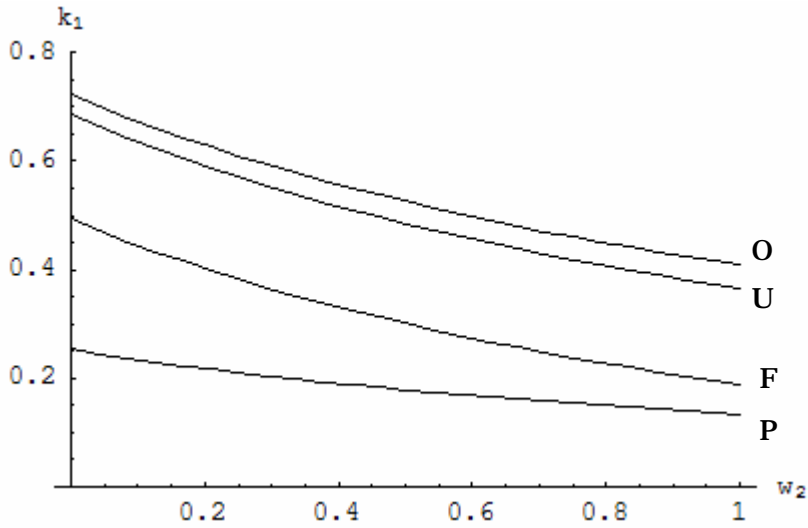
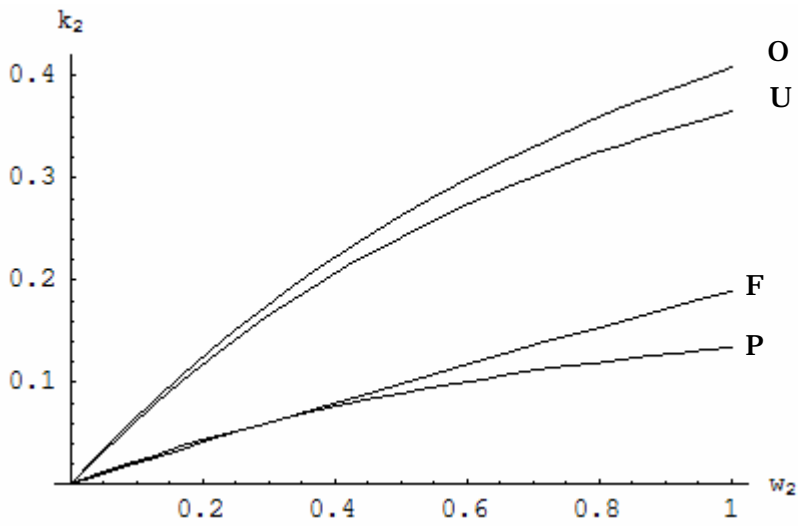


Figure 2 Effects of the difference in the wage rate

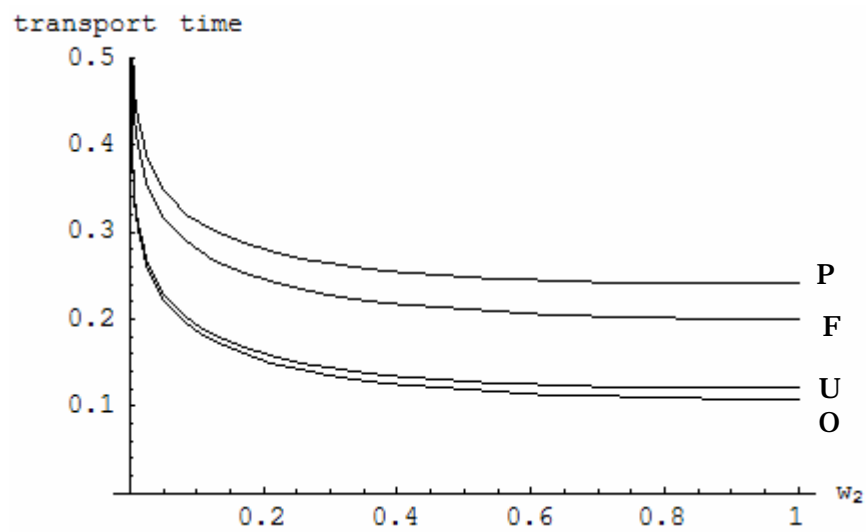
(a) Investment in country 1



(b) Investment in country 2



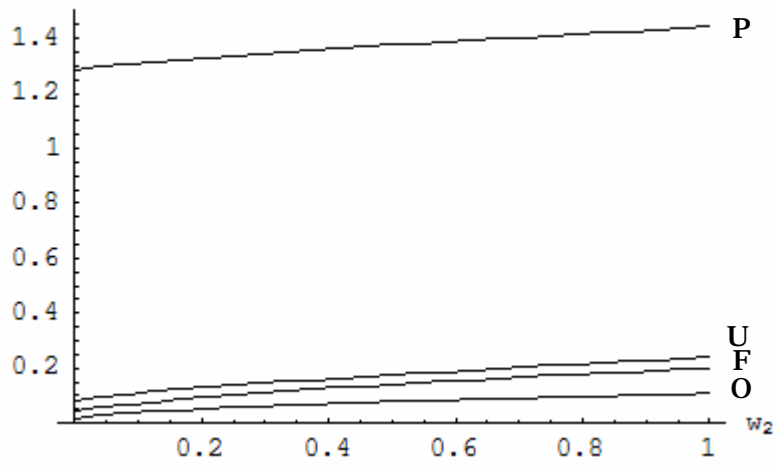
(c) Transport time



O: Optimum  
 F: Free access  
 P: Pricing and investment  
     by national government  
 U: User cost minimization

(d) Transport cost

transport cost



O: Optimum  
F: Free access  
P: Pricing and investment  
by national government  
U: User cost minimization

(e) Efficiency ratio

Efficiency ratio

