Time-limited land use rights and the Chinese urban housing market

中国都市における期限付土地使用権利と住宅市場に関する分析

Xuemei Jiang

Abstract

This paper develops a general spatial equilibrium model of a housing market with time-limited land use rights and analyzes the performance of the housing market in Chinese cities since tenure-housing policy was implemented. We interpret the reasons for the implementation of an affordable housing policy. Under an immature housing market, "housing construction" policy is the only means to improve the residential circumstances of low-income households. Land rent in upscale residential areas is lower than that of lower-scale residential areas; a subsidy policy to developers plays a more effective role when it induces lower income households to be located in higher land rent areas to consume more housing services. Empirical analyses are used to elucidate effects of affordable housing policy and population growth on increased housing prices.

Keywords: *commodity housing, affordable housing, time-limited right of land use, housing quality, land rent, maintenance, subsidy to developer*

1 Introduction

During the last 40 years, numerous studies have advanced models of competitive housing markets. Sweeny (1974), Braid (1986), and Arnott et al. (1997) analyze non-spatial, stationary-state filtering models of the housing market in which the quality deterioration of a housing unit depends on the endogenous level of maintenance. Hardman et al. (1999) create an overlapping-generation model in continuous time with the assumption of immutable and indestructible housing. These analyses ignore land and the spatiality of housing markets. Bruckner (1981) and Sasaki (1990) construct spatial models with durable housing. In those papers, the developer is cast as the property owner, and the property rights of land are infinite; maintenance is not considered.

The object of this study is to develop a general equilibrium spatial model of a housing market with time-limited land use right and analyze the housing market mechanism in urban China since tenure-housing policy has been implemented. This model posits a construction-demolition cycle of housing, and quality deterioration of a housing unit depends on the endogenous level of maintenance.

Time-limited land use rights will not constrain the efficiency of the competitive housing market. Miceli et al. (2005) provides an economic justification for time-limited property rights by arguing that they actually enhance property values in the presence of various sorts of market failure.

In China, urban land is owned entirely by the Chinese government (the State), and the rights of land use are allocated in the urban land market as goods. The State monopolizes land provision and supplies the rights of land use to the first-grade land market, either in the form of subletting (*churang*[1]) or in the form of a transfer (*huabo*[2]). The respective sublet periods of residential land, industrial land, and commercial land are 70 years, 50 years, and 40 years.

However, the government freely withdraws the right of using land from a developer if a developer does not start projects in two years according to sublet contracts. Therefore, bankers often evade risk and are reluctant to provide loans to developers subject to the right of land use at the stage of obtaining it. Therefore, developers gather lump-sum paid sublet fees independently; consequently, the rate of fundraising in real estate investment funds is large in China, as Table 1-1 shows. In addition, all sublet fees are lump-sum paid and included in the government fiscal revenue. They are used to construct fundamental municipal infrastructure and develop

Table 1-2 Fund sources of national real estate development investment

1,000,000 RMB YUAN

ITEM YEAR	1997	1998	1999	2000	2001
A) Total Funds	381,707	441,494	479,590	599,763	769,639
B) Domestic Loans	91,119	105,317	111,157	138,508	169,220
C) Fundraising	97,288	116,698	134,462	161,421	218,396
D) Rate of Funds (C/A) (%)	25.49	26.43	28.04	26.91	28.38

From China Real Estate Market Year Book 2001–2002

[1] Subletting is defined as a behavior by which the State, as the owner of the land, lends the right of land use (land lease) for a period; the land user pays a sublet fee (rent).

[2] Transfer is defined as a behavior by which the government beyond the prefectural level permits free land use after paying compensation according to legal procedures, or a transfer of the right of free land use. The transfer system is operated on behalf of the public welfare.

land. These characteristics are considered in this paper by description as behaviors of developers and the government.

Developers construct commodity housing on sublet residential land and affordable housing on transferred land. The government locates commodity housing mostly in the inner city, and affordable housing mostly on the outskirts of the city to obtain more land rental revenue. High-income households typically purchase or rent commodity housing, moderate and low-income households purchase affordable housing (*jingjishiyongfang*); lowest-income households rent inexpensive rental housing (*lianzufang*). The former two housing policies can be designated as *tenure housing* policies to promote the purchase of owner-occupied housing. The differences of the two are distinguishable according to the location and housing rent.

According to these characteristics, we construct a spatial model with a monocentric city concept. In the model, we presume that there are only residential buildings and that the city size is constant. The latter assumption is reasonable because the land resource is limited and the government operates public goods and distributes housing rent subsidy to low-income households with limited land rent revenue. In addition, Fujita (1982) presents that sprawl development occurs on equilibrium spatial growth paths only when multiple building types and multiple activity types exist (i.e., different types of households and/or different types of urban firms). This paper specifically describes only the case of residential buildings; the lot size of each housing unit equals one. For those reasons, the constant city size assumption is reasonable.

This study extends Jiang (2006), develops dynamic models and theoretically analyzes construction-demolition cycle housing markets with time-limited land use rights. Moreover, it also executes empirical analysis using urban data of 27 Chinese provinces and 4 autonomous municipalities to elaborate Chinese urban housing market status and verify theoretical results.

In section 2, behaviors of developers, consumers and the government are described. Section 3 presents a general equilibrium, section 4 presents results of empirical analyses with Chinese urban data; section 5 summarizes the results.

2 Assumptions and behaviors

2.1 Developer behavior

The framework of this model adopts the concept of Jiang (2006): commodity housing located in the inner monocentric linear city lying on a featureless plain and containing a central business district (CBD) of zero area at the origin, with affordable housing located on the outskirts of the city.

All residential buildings are assumed to deteriorate over time; their lives are finite. Therefore, a construction-demolition cycle of housing pertains with assumed zero demolition cost. In addition, all housing has one unit floor space; its service is determined by its quality ($q(t)$), which is derived from its construction quality ($q0$), maintenance technology ($f(m,q)$) and maintenance expenditure ($m(t)$).

In addition, the following assumptions are made:

The government owns all land of a city and leases land to residents; it determines the period of land use right (T) (in another words, the land lease period), and requires a sublet fee (land rent) ($L(T,d)$) to be paid by lump sum. The lowest level of lump-sum land rent for T period is not less than the agricultural land rent ($RA(T)$) for T period: $L(T,d) \geq RA(T)$. Commodity housing therefore comes to be located in the zone from the CBD to the inner city boundary (D_1) while affordable housing comes to be located from the inner city boundary to the outer city boundary (D_2), and the city size (D_2) is constant. The land rent of affordable housing is zero because its land is transferred. Affordable housing rent is constrained as the remainder of $(R(q) - \frac{1}{b(d)} l(t,d))$, where $R(q)$ and $\frac{1}{b(d)} l(t,d)$ respectively represent the market housing rent and the land rent of location d per unit housing. Furthermore, the government controls the construction density ($b(d)$) through zoning; all construction densities are planned as equal in the same zoning period.

No differences pertain between owners and renters of housing. All households request developers to operate their housing. Developers have perfect foresight and execute optimal operational programs. Initially, they obtain the use right of residential land for T period at location d by paying a lump-sum sublet fee ($L(T,d)$) (in other words, lump-sum paid land rent), then choose construction quality ($q0$), and the time path of maintenance expenditure $(m(t))$ over the building's operating life (S) to maximize their profits, with a market-determined housing rent $(R(q))$.

The profit of commodity housing per unit land area is defined as Eq. (2-1-1-A).

$$\pi = b(d)\int_0^S [R(q(t)) - m(t)]e^{-rt} dt - L(T,d) - C(b,q0) \qquad \text{(2-1-1-A)}$$

$$s.t. \quad \dot{q} = f(m,q)$$

$$m(t) \geq 0$$

$$q(t) > 0$$

$$L(T,d) \geq RA(T)$$

where S represents the operating period of commodity housing, and is equivalent to T under a maximized profit condition (Refer to *Appendix 1*). In addition, the following are given:

d is the distance from the CBD;

$b(d)$ is the construction density at location d, and $b(d) \geq 0$;

$q(t)$ is the housing quality at time t;

$R(q)$ is the housing rent, and $R_q > 0$, $R_{qq} = 0$ *;

$m(t)$ is the maintenance expenditure at time t;

r is the discount rate; $l(t,d)$ is the land rent at location d at time t; and

T is the land-lease period.

In addition, $L(T,d)$ is the lump-sum land rent paid for location d for T period at time 0; and $L(T,d) = \int_0^T e^{-rt} l(t,d) dt = T \cdot \bar{l}(d)$.

Also, $C(b,q0)$ is the construction cost for initial quality $(q0)$ with structural density b, and $\frac{\partial C}{\partial q} > 0, \frac{\partial C}{\partial b} > 0$.

Furthermore, $q0$ is the construction quality of commodity housing.

Also, $f(m,q)$ is the maintenance technology, and $f_m > 0$, $f_{mm} < 0$ (maintenance technology is convex), $f(0,q) < 0$ (buildings deteriorate with time), $f_q < 0$, $f_{qq} < 0$, and $f_{mm} f_{qq} - f_{mq}^2 \geq 0$ ($f(m,q)$ is concave).

* The partial derivative of a function with respect to a variable is denoted by a subscript.

Finally, $RA(T)$ is the agricultural land rent d for T period at time 0, and $RA(T)=\int_0^T e^{-rt} ra(t)dt = T\cdot\overline{ra}$.

Developers operate commodity housing to maximize Eq. (2-1-1-A): their profits. In Eq. (2-1-1-A), the first term of right-hand-side describes the operating income; the second term of right side is the lump-sum land rent.

The profit of affordable housing is defined as Eq. (2-1-1-B)**.

$$\pi^a = b(d)\int_0^{S^a}[R(q)-\frac{1}{b(d)}l(t,d)-m(t)]e^{-rt}dt+\int_0^{S^a} i(t)L(T,d)e^{-rt}dt - C(b,q0^a)$$

$$= b(d)\int_0^{S^a}[R(q)-m(t)]e^{-rt}dt-\int_0^{S^a} l(t,d)e^{-rt}dt+\int_0^{S^a} i(t)L(T,d)e^{-rt}dt - C(b,q0^a)$$

(2-1-1-B)

$$s.t.\quad \dot{q} = f^a(m,q)$$

$$m(t)\geq 0$$

$$q(t)>0$$

where S^a is the operating period of affordable housing, $f^a(m,q)$ is maintenance technology and concave ($f^a{}_m>0$, $f^a{}_{mm}<0$, $f^a(0,q)<0$, $f^a{}_q<0$ and $f^a{}_{qq}<0$), $q0^a$ is the construction quality of affordable housing. Furthermore, $i(t)$ represents the rate of capital return at time t.

Developers can construct affordable housing with a smaller capital investment in contrast to commodity housing because of the assumed zero cost of land use. Moreover, they can earn a capital return with the saved investment capital ($L(T,d)$) from other capital markets, as described as the second term of right side in Eq. (2-1-1-B). Therefore, when the rate of capital return from the other capital market is large,

** Superscript (a) stands for affordable housing.

developers are eager to develop affordable housing, although its operating income (the first term of right side in Eq. (2-1-1-B)) is lower than that of commodity housing. They operate affordable housing to maximize Eq. (2-1-1-B).

Current value Hamiltonians for Eqs. (2-1-1-A) and (2-1-1-B) with respect to m are posed respectively, as

$$H = b(d)[R(q(t)) - m(t)] + \lambda(t) f(q,m) \tag{2-1-2-A}$$

$$H^a = b(d)[R(q(t)) - m(t)] + \lambda^a(t) f^a(q,m) \tag{2-1-2-B}$$

where $\lambda(t)$ and $\lambda^a(t)$ are adjoint variables and the mean marginal benefit per quality change of commodity housing and affordable housing separately by maintenance.

According to the Pontryagin maximum principle, the following canonical equations can be written except at points of discontinuity of m^* and m^{a^*} (the assumptions of $f_{mm} < 0$ and $f^a{}_{mm} < 0$ provide that there exist unique m^* and m^{a^*} for each (q,λ) and (q^a,λ^a)). In addition, we treat Eqs. (2-1-2-A) and (2-1-2-B) as maximized Hamiltonians.

State equations:

$$H_\lambda = \frac{\partial H}{\partial \lambda} = f(q,m) \tag{2-1-3-A}$$

$$H^a{}_{\lambda^a} = \frac{\partial H^a}{\partial \lambda^a} = f^a(q,m) \tag{2-1-3-B}$$

Multiplier equations:

$$\dot{\lambda}(t) = \frac{\partial \lambda}{\partial t} = r\lambda - \frac{\partial H}{\partial q} = r\lambda - b(d)R_q - \lambda f_q \tag{2-1-4-A}$$

$$\dot{\lambda}^a(t) = \frac{\partial \lambda^a}{\partial t} = r\lambda^a - \frac{\partial H^a}{\partial q} = r\lambda^a - b(d)R_q - \lambda^a f^a{}_q \tag{2-1-4-B}$$

Optimality conditions:

$$H_m = \frac{\partial H}{\partial m} = -b(d) + \lambda f_m = 0 \qquad \text{(2-1-5-A)}$$

$$H^a{}_m = \frac{\partial H^a}{\partial m} = -b(d) + \lambda^a f^a{}_m = 0 \qquad \text{(2-1-5-B)}$$

From the above, $\lambda = \frac{b(d)}{f_m} \geq 0$ and $\lambda^a = \frac{b(d)}{f^a{}_m} \geq 0$ are derived with assumptions $b(d) \geq 0$, $f_m > 0$ and $f^a{}_m > 0$. Furthermore, we can obtain the results of $H_{mm} = \lambda f_{mm} \leq 0$ and $H^a{}_{mm} = \lambda^a f^a{}_{mm} \leq 0$ with assumptions $f_{mm} < 0$ and $f^a{}_{mm} < 0$, which meet necessary conditions for maximization problems. In addition, because of the concavity of $f(m,q)$ and $f^a(m,q)$, both $\lambda \geq 0$ and $\lambda^a \geq 0$ are sufficient conditions for optimality.

From boundary and transversality conditions, we derive the following relations.

$$\lambda(0) = \frac{\partial C(b,q0)}{\partial q0} \qquad \text{(2-1-6-A)}$$

$$\lambda^a(0) = \frac{\partial C(b,q0^a)}{\partial q0^a} \qquad \text{(2-1-6-B)}$$

$$H(S) = e^{rT}\bar{l}(d) \qquad \text{(2-1-7-A)}$$

$$H^a(S^a) = l(S^a,d) - i(S^a)L(T,d) \qquad \text{(2-1-7-B)}$$

The multipliers $\lambda(0)$ and $\lambda^a(0)$ are the marginal valuations of $q0$ and $q0^a$ separately at time 0. Therefore, Eqs. (2-1-6-A) and (2-1-6-B) imply that construction occurs only when the marginal benefit equals the marginal cost of quality.

The maximized profit functions of commodity housing and affordable housing can be rewritten using the preceding equations, separately, as

$$\pi^* = b(d)\int_0^S [R(q(t)) - m(t)]e^{-rt}dt - L(T,d) - C(b,q0)$$
$$= \int_0^S (H - \lambda f)e^{-rt}dt - L(T,d) - C(b,q0) \qquad \text{(2-1-8-A)}$$
$$= \frac{1}{r}H(0) - (1 + \frac{1}{rT})L(T,d) - C(b,q0)$$

$$\pi^{a^*} = b(d)\int_0^{S^a} [R(q) - \frac{1}{b(d)}l(t,d) - m(t)]e^{-rt}dt + \int_0^{S^a} i(t)L(T,d)e^{-rt}dt - C(b,q0^a)$$
$$= \int_0^{S^a} (H^a - \lambda^a f^a)e^{-rt}dt + \int_0^{S^a} [i(t)L(T,d) - l(t,d)]e^{-rt}dt - C(b,q0^a)$$
$$= \frac{1}{r}[H^a(0) - e^{-rS^a}H^a(S^a)] + \int_0^{S^a} [i(t)L(T,d) - l(t,d)]e^{-rt}dt - C(b,q0^a)$$
$$= \frac{1}{r}[H^a(0) + i(0)L(T,d) - l(0,d)] + \frac{1}{r}\int_0^{S^a} [\dot{i}(t)L(T,d) - \dot{l}(t,d)]e^{-rt}dt - C(b,q0^a)$$

(2-1-8-B)

where $\dot{i}(t) = \frac{\partial i(t)}{\partial t}$ and $\dot{l}(t,d) = \frac{\partial l(t,d)}{\partial t}$.

The maintenance technology of commodity housing and affordable housing are assumed to be identical, and m is eliminated to consider both the commodity housing operating program and the affordable housing operating program in the same $\lambda - q$ plane.

From Eqs. (2-1-4-A) and (2-1-4-B),

$$\lambda = \frac{bR_q}{r - f_q} \ (\dot{\lambda} = 0). \qquad \text{(2-1-9)}$$

Because $R_{qq} = 0$ and $f_{qq} < 0$, this is a decreasing function of q.

Using Eqs. (2-1-3-A), (2-1-3-B), (2-1-4-A), (2-1-4-B), (2-1-5-A), and (2-1-5-B), the $\dot{q} = 0$ locus can be sketched. We assume that the curve is upward-sloping and that it intersects with the $\dot{\lambda} = 0$ locus at X (where $\dot{\lambda} = 0$ and $\dot{q} = 0$) above the marginal construction cost ($\frac{\partial C}{\partial q}$) locus, as depicted in Fig. 2-1 (it is proved in lemma 4). This pattern implies that the marginal cost of quality via maintenance is greater than the

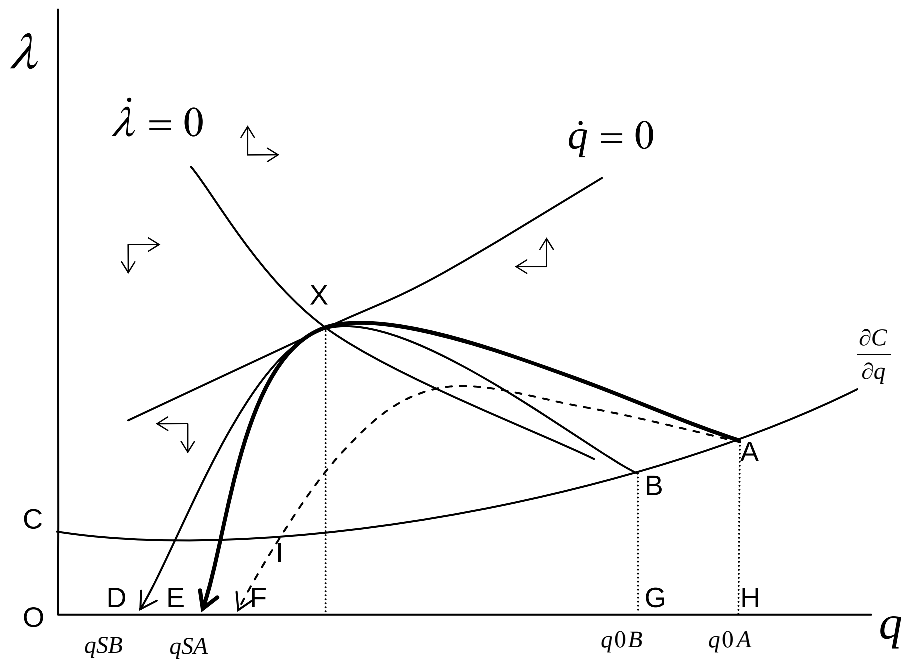

Fig. 2-1 Phase diagram for housing operation

marginal cost of quality via construction.

Under the construction-demolition cycle, all housing units are constructed when the marginal benefit equals to marginal construction cost of quality ($\lambda(0) = \dfrac{\partial C(b, q0)}{\partial q0}$ and $\lambda^a(0) = \dfrac{\partial C(b, q0^a)}{\partial q0^a}$), and that housing units are destroyed when the marginal benefit equals zero ($\lambda(S) = 0$ and $\lambda^a(S^a) = 0$). Moreover, operating trajectories of commodity housing always pass the intersection X because those paths maximize profits. The operating revenue with path AXE is greater than that of path AIF with identical construction costs, as presented in Fig. 2-1.

According to the phase diagram analysis, we can rewrite the profit functions of Eqs. (2-1-8-A) and (2-1-8-B) as the following.

$$\pi^* = \int_{qS}^{q0} \lambda dq - \int_{0}^{q0} \frac{\partial C}{\partial q} dq - L(T, d) = \int_{qS}^{q0} \lambda dq - C(b, q0) - L(T, d) \qquad \text{(2-1-10-A)}$$

$$\pi^{a^*} = \int_{qS^a}^{q0} \lambda dq - \int_0^{q0} \frac{\partial C}{\partial q} dq + \int_0^{S^a} [i(t)L(T,d) - l(t,d)]e^{-rt} dt$$
$$= \int_{qS^a}^{q0} \lambda dq - C(b,q0) + \int_0^{S^a} [i(t)L(T,d) - l(t,d)]e^{-rt} dt \qquad (2\text{-}1\text{-}10\text{-}B)$$

Therein, qS is the terminal quality of commodity housing, and qS^a is the terminal quality of affordable housing.

2.2 Government behavior

The government owns all land in a city. The government simultaneously leases all land to residents for commodity housing for T period. The important role of the government is to determine housing policies and the strategy of the public goods management.

Affordable housing policy is implemented at the constructional stage; its subsidy per affordable housing land unit is distributed directly to a developer at the amount of $[i(t)L(T,d)]$ at time t (the total amount is $[\int_0^{S^a} i(t)L(T,d)e^{-rt}dt]$)* . Therefore, we abbreviate it as developer subsidy shown below.

The present value of total revenue from land rent for T period at time 0 is

$$TL(T) = \int_0^{D_1} L(T,d)dd \qquad (2\text{-}2\text{-}1)$$

where D_1 and $L(T,d)$ respectively denote the inner city boundary and lump-sum land rent for T period at time 0.

The government manages public goods and implements housing policies such that the revenue balances with expenditures.

---

* If we assume that the term of Eq. (2-1-1-B) $[\int_0^{S^a} l(t, d)e^{-rt}dt]$ equals $L(T, d)$, in contrast to Eq. (2-1-1-A), it is readily understood that $[\int_0^{S^a} i(t)L(T,d)e^{-rt}dt]$ is the total subsidy distributed to a developer.

$$TL(T) = \int_0^T gG(t)e^{-rt}dt + \int_0^T k(t)e^{-rt}dt \quad (2\text{-}2\text{-}2)$$

$$\text{s.t. } G(t) = G0 + \int_0^t \dot{G}dt$$

$$\dot{G} = \psi(k(t))$$

Therein, $G0$ represents the initial public goods service at time 0,

g is the constant operating cost per public good service,

$k(t)$ is the investment to new public goods provision at time t,

$\psi(k(t))$ is the production technology of public goods that will not be improved, and $\psi_k > 0$, $\psi_{kk} < 0$ and $\psi(k) >= 0$.

2.3 Consumer behavior

We presume that each household with identical well-behaved preference is myopic at each time that the entire household income is used for the composite goods and housing service to achieve the household's expected utility. In addition, composite goods and housing services are normal goods.

Presuming that continuous different income levels ($\{y_j\}$) and $y_{\min} \le y_j \le y_{\max}$, exist, and that the government divides the total urban population, N, into high-income ($y^h \in [y_j \ge \bar{y}]$) and low-income ($y^l \in [y_j < \bar{y}]$) groups by setting a threshold income ($\bar{y}$), such that $N = N^h + N^l$, where N^h is the number of high-income households and N^l is that of low-income households.

The utility level of high-income households who reside in commodity housing at time t is described as Eq. (2-3-1).

$$\underset{\{x,q\}}{Max} \quad U(x(t), q(t), G(t)) = V(t, y^h) \quad (2\text{-}3\text{-}1)$$

$$s.t. \quad y^h(t) = x(t) + R(q)$$

The utility level of low-income households with developer subsidy at time t, who reside in affordable housing, is shown as Eq. (2-3-2).

$$\underset{\{x,q\}}{Max} \quad U(x(t),q(t),G(t)) = V(t,y^l) \qquad (2\text{-}3\text{-}2)$$

$$s.t. \quad y^l(t) = x(t) + (R(q) - \frac{l(t,d)}{b(d)})$$

In that equation, $U(x,q,G)$ is the utility function ($U_x > 0$, $U_q > 0$ and $U_G > 0$).

$V(y_j(t),t)$ is the expected utility level of households of income $y_j(t)$ at time t.

$x(t)$ is the consumption of composite goods, which price is numeraire.

$q(t)$ is the housing service at time t.

$G(t)$ is the public goods service at time t.

$y_j(t)$ is the income level at time t.

Finally, $R(q)$ is the housing rent.

3 Equilibrium in a constant environment with a subsidy given to the developer

For this equilibrium, we presume a constant environment for which there are no changes on either income, or population, or public goods service, and residents do not move in a city either into another city: they live in the same housing units for a construction-demolition cycle life.

3.1 The land-market equilibrium

In the housing market with free entry and exit, developers' profits are zero. Therefore, the lump-sum land rent $L^*(T,d)$ that makes developer's profit zero is the highest willingness-to-pay of developers and is the land rent in equilibrium. In reality, land for commodity housing will be sublet to the developer offering to pay the highest sublet fee. The developer rents the land with commodity housing to the high-income household offering the highest bid-rent. Using Eq. (2-1-8-A), it is derived as

$$L^*(T,d) = \frac{rT}{1+rT}[\frac{1}{r}H(0) - C(b,q0)] \cdot \quad (3\text{-}1)$$

Lemma 1: $\frac{\partial L^*(T,d)}{\partial T} > 0$ (3-2)

That inequality represents that land rent is higher with a longer land-lease period.

The government only distributes developer subsidy and maintains initial public goods service through T period such that the revenue balances with expenditure. From Eqs. (2-2-1) and (2-2-2),

$$\int_0^{D_1} L^*(T,d)dd = \int_0^T gG_0 e^{-rt} dt \,. \quad (3\text{-}3)$$

3.2 The housing market equilibrium

High-income residents reside in commodity housing and low-income households live in affordable housing. Therefore, the number of commodity housing units and affordable housing units respectively equal to the number of high-income households and low-income households, as described in Eqs. (3-4) and (3-5).

$$\int_0^{D_1} b(d)dd = N^h \quad (3\text{-}4)$$

$$\int_{D_1}^{D_2} b(d)dd = N^l \quad (3\text{-}5)$$

Residents choose a housing service level to maximize their respective utilities at each time. Moreover, developers operate commodity housing and affordable housing according to the demand-side needs. Therefore, they determine the initial construction quality level ($q0$) according to Eqs. (3-6-A) and (3-6-B). With the deterioration of housing quality, residents' utilities are decreasing with time because of $R_q > 0$ and $R_{qq} = 0$.

$$MRS(y^h - R(q), q0) = -R_q \quad (3\text{-}6\text{-}A)$$

$$MRS(y^l - R(q) + {}^{l}\!/_{b}, q0^a) = -R_q \qquad (3\text{-}6\text{-}B)$$

Lemma 2: $\dfrac{\partial y^h}{\partial q0} = x_{q0} + R_q > 0$ (3-7)

Proof: Solving the maximization problem of Eq. (2-3-1), the consumption of composite goods and initial housing service are derived; the roots are determined according to the income level and the utility function type. Therefore, we can respectively describe consumption of composite goods and initial housing service as functions of income such as $x = x(y^h)$ and $q0 = q0(y^h)$. In addition, the function $q0 = q0(y^h)$ can be inverted as $y^h = q0^{-1}(q0)$; then consumption of composite goods is presented as a function of consumption of initial housing service: $x = x(y^h) = x(q0^{-1}(q0))$. Thus, $x_q = \dfrac{\partial x}{\partial y^h}\dfrac{\partial y^h}{\partial q0} > 0$, because composite goods (x) and initial housing service ($q0$) are

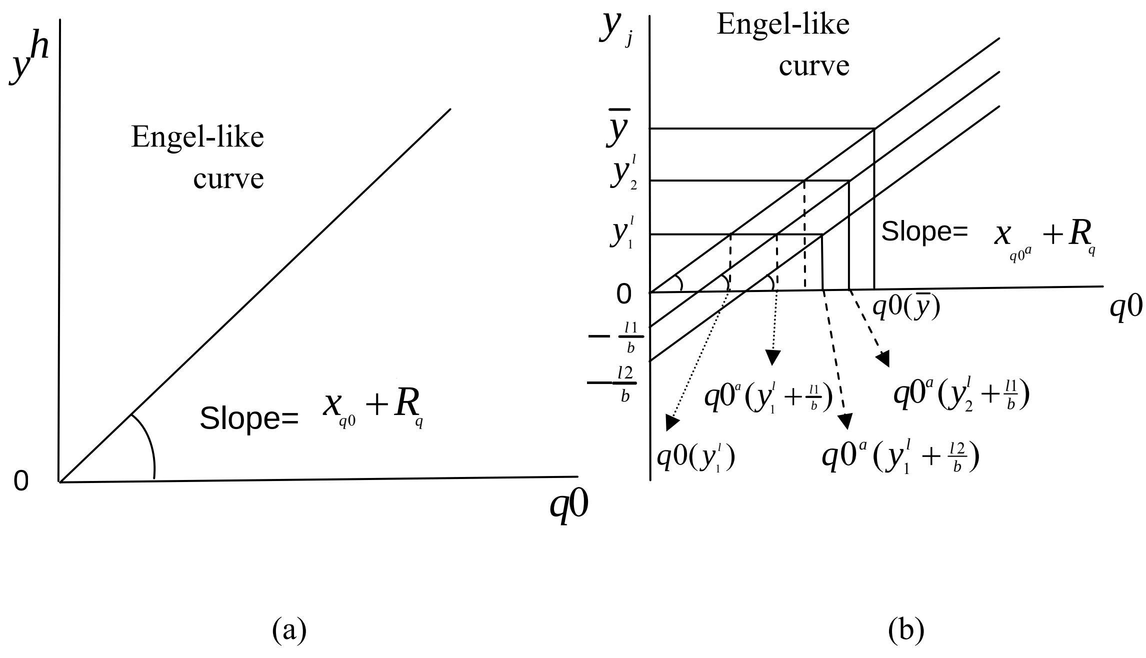

Fig. 3-1 Engel-like curve

normal goods ($\frac{\partial y^h}{\partial x} > 0$ and $\frac{\partial y^h}{\partial q0} > 0$). Therefore, Eq. (3-7) is satisfied with the assumption of $R_q > 0$.

Higher income households choose greater initial housing service. As presented in Fig. 3-1-a, the curve is similar to an Engel curve (called an Engel-like curve hereafter). Developers construct commodity housing at a continuous quality level along the Engel-like curve according to household needs with given continuous income levels.

Lemma 3: $\frac{\partial (y^l + \frac{l(t,d)}{b})}{\partial q0^a} = x_{q0^a} + R_q > 0$ (3-8)

Equation (3-8) can be derived using a similar method to that presented for Eq. (3-7). Construction quality of affordable housing is not certainly continuous because it also is decided by location land rent ($l(t,d)$), as presented in Fig. 3-1-b. For low-income households with income y_1^l, their initial housing services are $q0^a(y_1^l + \frac{l1}{b})$ when they locate the area with land rent $l1$; their initial housing service is equivalent to $q0^a(y_1^l + \frac{l2}{b})$ when they locate in an area with land rent $l2$. Because $l2$ is greater than $l1$, $q0^a(y_1^l + \frac{l2}{b}) > q0^a(y_1^l + \frac{l1}{b})$, which means that residential status will improve better when lower income households locate higher land rent areas.

In addition, the developer subsidy policy will play more effective role as they let lower income households locate in higher land-rent areas to consume better housing services.

Lemma 4: $\frac{bR_{q0(y^h)}}{r - f_{q0(y^h)}} \leq \frac{\partial C(q0(y^h))}{\partial q0(y^h)} < \frac{bR_{qX}}{r - f_{qX}}$ for commodity housing (3-9)

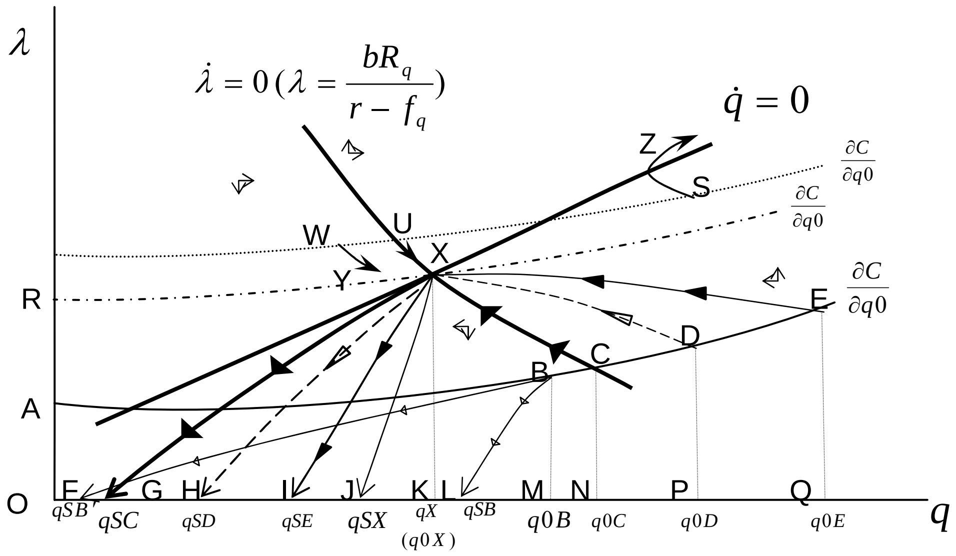

Fig. 3-2 Phase diagram

The marginal construction cost ($\frac{\partial C}{\partial q}$) locus lies below intersection X (qX is the quality at intersection X) of the $\dot{\lambda} = 0$ locus and the $\dot{q} = 0$ locus in Fig. 3-2, namely the marginal cost of quality via maintenance is greater than the marginal cost of quality via construction. Construction occurs when the marginal construction cost ($\frac{\partial C}{\partial q}$) locus crosses and lies on the right side of the $\dot{\lambda} = 0$ locus (the area of CE line).

Proof: i) If $\frac{bR_{q0(y^h)}}{r - f_{q0(y^h)}} > \frac{\partial C(q0(y^h))}{\partial q0(y^h)}$ (the left side of the $\dot{\lambda} = 0$ locus), the operating profit of commodity housing will be negative and no developer will enter the project operating when all housing quality deteriorates with time. For example, as presented in Fig. 3-2, a housing unit with quality qoB (at point B, $\frac{bR_{q0}}{r - f_{q0}} > \frac{\partial C}{\partial q0}$ is satisfied) is developed and destroyed at quality qSB; consequently, its operating profit is negative because the operating revenue ($\Delta\ MBL$) is less than the construction cost ($\square OMBA$).Therefore, the condition $\frac{bR_{q0}}{r-f_{q0}} > \frac{\partial C}{\partial q0}$ is rejected in the case of commodity housing operations.

Proof: ii) If $\frac{\partial C(q0(y^h))}{\partial q0(y^h)} = \frac{bR_{qX}}{r-f_{qX}}$ (at intersection X), the operating profit of commodity housing will be negative when all housing quality deteriorates over time. As presented in Fig. 3-2, the only possible operating trajectory of a housing unit with construction quality $q0X$ (at point X, $\frac{\partial C}{\partial q0} = \frac{bR_{qX}}{r-f_{qX}}$ is satisfied) is XJ; and its operating profit is negative because the operating revenue ($\triangle KXJ$) is less than the construction cost ($\square OKXR$). Therefore, the condition $\frac{\partial C}{\partial q0} = \frac{bR_{qX}}{r-f_{qX}}$ is rejected in the case of commodity housing operations also.

Proof: iii) If $\frac{\partial C(q0(y^h))}{\partial q0(y^h)} > \frac{bR_{qX}}{r-f_{qX}}$ (above the intersection X), all possible operating trajectories, for example, paths UX, SZ, and WY have quality upgrading, as presented in Fig. 3-2. It contradicts our assumption that all housing quality deteriorates with time. Therefore, the condition $\frac{\partial C}{\partial q0} > \frac{bR_{qX}}{r-f_{qX}}$ is dismissed.

Lemma 5: $\frac{\partial L^*(T)}{\partial y^h} < 0$ and $\frac{\partial L^*(T)}{\partial q0(y^h)} < 0$ (3-10)

This is derived from Eqs. (3-2), (3-7) and (3-9); it means that higher income households come to be located in lower land rent areas. In addition, it implies that land rent of an upscale residential area is less than that of the lower-class residential area. The result is interesting and coincides with the urban economy theorem, even though transport costs are not considered.

Lemma 6: $L^*(T)_{\min} = \int_{qS}^{q0(y_{\max})} \lambda dq - C(b, q0(y_{\max})) \geq RA(T)$ (3-11)

$$L^*(T)_{\max} = \int_{qS}^{q0(\bar{y})} \lambda dq - C(b, q0(\bar{y})) \quad (3\text{-}12)$$

When $\frac{bR_{q0(y^h)}}{r - f_{q0(y^h)}} \leq \frac{\partial C(q0(y^h))}{\partial q0(y^h)} < \frac{bR_{qX}}{r - f_{qX}}$ and $\frac{\partial L^*(T)}{\partial q0(y^h)} < 0$, the construction quality level $q0(y_{\max})$ is the highest initial quality of commodity housing located at the lowest land rent, and $q0(\bar{y})$ is the lowest initial quality level of commodity housing located at the highest land rent. In addition, its operating trajectories can be described as paths EXI (from construction quality ($q0E$) to demolition quality (qSE)) and CXG (from construction quality ($q0C = q0(\bar{y})$) to demolition quality (qSC)) in Fig. 3-2.

Developers will not construct housing with initial quality $q0 < q0(\bar{y})$ (quality lies on the left side of $q0C$ in Fig. 3-2) for low-income households because of negative profits. To some degree, it reflects the reason why the Chinese government distributes subsidies to developers to encourage their construction of affordable housing for low-income households for an immature housing market. In such an immature housing market, newly constructed housing is the only method to improve residential circumstances for low-income households.

It also implies that some time highest-income residents cannot consume housing services commensurate with their incomes. Developers will not construct housing with initial quality $q0 > q0(y_{\max})$ (quality lies on the left side of $q0E$ in Fig. 3-2) for residents whose income is greater than $y_{\max}$, when $L^*(T)_{\min} = RA(T)$, because of negative profits.

Lemma 7: $\frac{\partial C(q0(y^l))}{\partial q0(y^l)} < \frac{bR_{q0(y^l)}}{r - f_{q0(y^l)}}$ for affordable housing (3-13)

In equilibrium, profits of all developers are zero. Therefore, the quality of affordable housing must be less than $q0(\bar{y})$ and land rent ($\frac{l(t)}{b}$), especially $\frac{ra(t)}{b}$, is very small, which is related directly to the developer subsidy. As discussed for lemma 4, the operating profit is negative when $\frac{\partial C(q0)}{\partial q0} < \frac{bR_{q0}}{r - f_{q0}}$ and all housing quality deteriorates over time, so the government must distribute subsidies to encourage developers to construct low-quality housing for low-income households. In Fig. 3-2, the initial housing quality levels on the left side of $q0C$ ($q0(\bar{y})$) are for low-income households.

If not, in other words, when $q0(y^l + \frac{l(t,d)}{b}) \geq q0(\bar{y})$ (affordable housing quality level lies on the right side of $q0C$ in Fig. 3-2), profits of these housing units are significantly positive. In this case, the government can not receive land rent revenue: the land rent handed back to consumers is zero, so consumers improve their utilities only slightly in contrast to developers. Mostly benefited are the developers, they gain not only operating profits of affordable housing, which equals the land rent of corresponding commodity housing, but also another capital returns.

To some extent, it reflects the actual reason why developers reap large profits through newly constructed housing for low-income households, which causes inequality distribution of resource. It is a trap of affordable housing policy in reality, but it is not easy to overcome because the government has difficulty determining the benchmark of the threshold income and developer subsidy; it merely imposes the housing policy that most of low-income households can improve their residential circumstances.

It is better to let low income households consume filtering-down commodity housing with consumer subsidies than to let them consume newly constructed affordable housing when the initial quality level of affordable housing is less than qSE in the mature market in Fig. 3-2.

Lemma 8: $\frac{\partial L^*(T)}{\partial q0^a(y^l)} < 0$ and $\frac{\partial L^*(T)}{\partial y^l} < 0$ (3-14)

It is derived from Eqs. (2-1-8-B), (3-8), and (3-13). Moreover, it implies that lower-income households locate in higher land-rent areas because the ratio of capital return is constant (refer to section 3.3), and the operating deficit of affordable housing is larger with lower initial quality. For that reason, developers need more subsidies to undertake the project.

In addition, it implies that the matched developer subsidy is distributed and that the developer subsidy policy is as effective as the government efforts. As discussed in relation to lemma 2, when lower income residents locate in higher land-rent areas, the subsidy policy to developers will be more effective.

Lemma 9: $\frac{\partial S^a}{\partial q0^a} > 0$, $\frac{\partial S^a}{\partial \bar{l}(d)} > 0$, $\frac{\partial S^a}{\partial i^*} > 0$ and $\frac{\partial S^a}{\partial r} < 0$ with the assumption of $e^{-rt}l(t,d) = l(0,d) = \bar{l}(d)$. (3-15)

We assume that $e^{-rt}l(t,d) = l(0,d) = \bar{l}(d)$ to simplify analysis; then from Eq. (2-1-8-B) the operating period of affordable housing is described as

$$S^a = \frac{1}{\bar{l}(d)}(\frac{1}{r}H^a(0) - C) + \frac{i^*}{r}T - \frac{1}{r} \quad \cdot \qquad (3\text{-}16)$$

It is easy to derive Eq. (3-15) from Eqs. (3-13) and (3-16).

The operating period of affordable housing becomes longer with either higher construction quality, or higher land rent, or a larger rate of capital return, or a lower discount rate. That is true because the operating deficit is smaller with a higher initial quality; developers obtain more capital revenue with either higher land rent or a larger rate of capital return. The present value of developers' profits at time 0 is greater when the discount rate is smaller. Therefore, they seek to operate affordable housing longer.

3.3 Financial market equilibrium

If the financial market is mature and efficient, the ratio of capital return $i(t)$ will be constant as i^*; it drives the profit of affordable housing to zero in equilibrium. Using Eq. (2-1-8-B), the rate of the capital return might be described as

$$i_j = \frac{1}{L^*(T,d)}[\int_0^{S^a} l^*(t,d)e^{-rt}dt + rC(b,q0^a(y_j)) - H^a(0,q0^a(y_j)) + \frac{e^{-rS^a}}{r} l^*(S^a,d)]$$

(3-17)

Lemma 10: $i^* = Max[i_j]$ (3-18)

The equilibrium capital return rate (i^*) must be sufficiently large that developers have incentives to construction affordable housing and to confirm that every low-income household has quality housing in which to reside.

Lemma 11: $\frac{\partial i^*}{\partial \bar{l}(d)} < 0$ and $\frac{\partial i^*}{\partial r} > 0$ with the assumption of $e^{-rt}l(t,d) = l(0,d) = \bar{l}(d)$.

(3-19)

This lemma is derived from Eq. (3-16).

Affordable housing comes to be located in low (high) land rent areas when i^* is very large (small). This paper implies a large capital return rate (i^*), with the assumption that affordable housing is located on the outskirts of a city.

In addition, with a higher discount rate (r), the rate of capital return (i^*) must be larger to maintain the present value of capital revenue at time 0 as positive.

Taken together, the results described above suggest that residential area formation is closely related to the income level.

4 Empirical analyses

4.1 Data

Data related to the Chinese urban housing market are collected from China Statistical Yearbooks (1999–2006) and China Real Estate Market Yearbooks (1999–2005). Urban data of 27 provinces and 4 autonomous municipalities (province-level cities) include average selling prices of residential buildings, average selling prices of affordable housing, floor space of residential buildings sold, floor space of affordable housing sold, the per-capita floor space of residential buildings, cost of buildings completed, average money wage, the nominal interest rates on loans, urban amenities, and so on.

In this analysis, population information plays a very important role as reflecting housing market demand; therefore, we apply urban population[*] data, although non-agriculturally registered population data are easily obtained. In addition, some non-agriculturally registered populations are not urban residents. Urban populations in 2000 and 2005 are compiled from China Statistical Yearbooks published in 2001 and

[*] Residents who reside habitually in urban area irrespective of their registrations.

2006. On the other hand, urban populations in 1998, 1999, 2001, 2002, 2003, and 2004 are inferred based on 2000 and 2005 Chinese National Population Census data. As Fig. 1-1 shows, the Chinese urbanization level has evolved from 33.25 in 1998 to 42.99 in 2005, thereby accelerating urban population growth.

Mortgage interest rates used in this paper are long-term (greater than 5 years) mortgage rates published by the People's Bank of China, and loan interest rates on developers are 120%[**] adjusted legal lending rates on capital construction investment loans, as published also by the People's Bank of China.

As Fig. 4-1 presents, the national average annual income has increased from 7,479 yuan in 1998 to 18,364 yuan in 2005 accompanying economic development. In addition, housing prices have risen from 1,807 yuan/ m^2 in 1998 to 2,937 yuan/ m^2 in 2005, even though the consumer price index has changed little. National and all other consumer price general indices are calculated by setting 1997 data as equal to 100, except for Tibet (for which 100 was set for 1998 data because of a lack of data). In relation to those, the per-capita housing floor space has not changed much.

Many reasons exist, such as housing prices soaring in some areas, interest rate changes, and geographical inequity between the east area and either the west or central areas. For this empirical analysis, we will do relative analysis.

[**] As of November 1998, the lending rate for small-sized enterprises could be 20% higher than nominal interest rates; as of September 1999, the lending rate for medium and small-sized enterprises could be 30% higher than nominal interest rates. (Both are published by the People's Bank of China.)

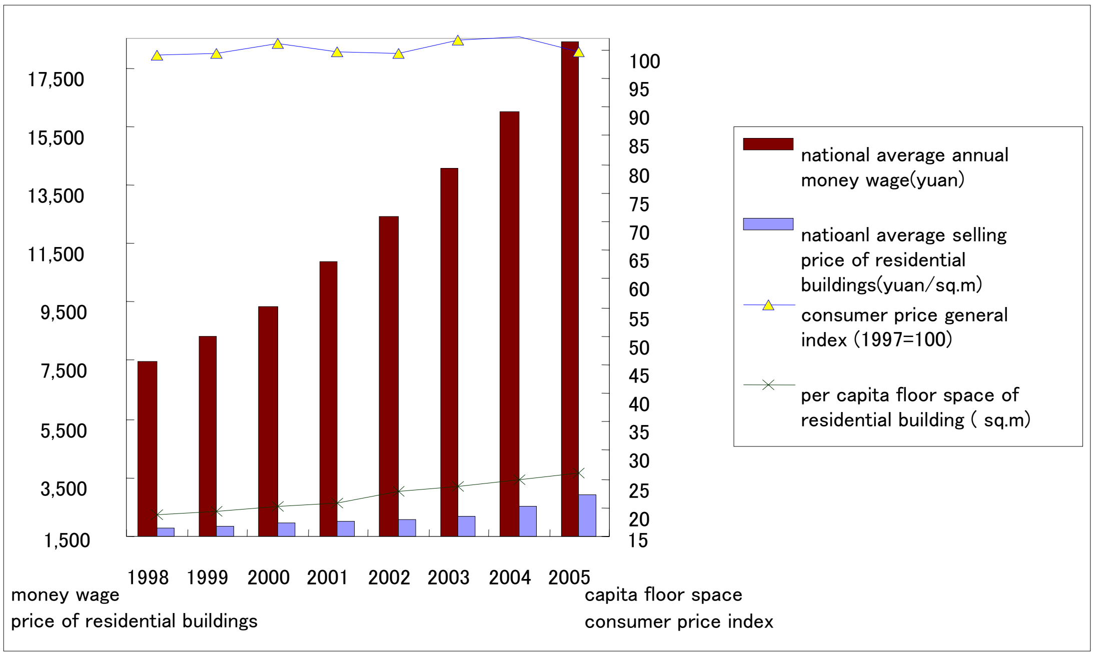

Fig. 4-1 National average annual income, average housing price,

per-capita housing floor space and consumer price index

4.2 Empirical model and variables

Because housing demand is determined by such factors as housing prices ($pricech$), individual income ($wage$), and the mortgage interest rate ($ilind$), we can write the demand function as

$$Q^D = F(pricech, wage, ilind).$$

Developers supply housing according to its price, building cost ($cost$), and interest rate on developer loans ($ilcom$); therefore the supply function can be written as

$$Q^S = F(pricech, \cos t, ilcom).$$

Under the equilibrium $Q^D = Q^S$, model 1 can be derived as Eq. (4-1).

Model 1:

$$lpricech = \alpha 1 \cdot lwage + \beta 1 \cdot l\cos t + \gamma 1 \cdot lilcom + \eta 1 \cdot lilind \qquad (4\text{-}1)$$

In that equation, $\alpha 1$, $\beta 1$, $\gamma 1$ and $\eta 1$ are coefficients. In addition, $lpricech$ is the logarithm of average selling price of residential buildings, $lwage$ is the logarithm of average annual money wage, $lilcom$ is the logarithm of nominal interest rates on developers' loans, and $lilind$ is the logarithm of nominal interest rates on individual account housing loans.

In model 2, we analyze the effect of affordable housing police ($afpolicy$) and the public reserve deposit system ($ilavc$) on housing consumption ($capspace$).

The Chinese government repealed the old welfare housing allocation system in 1998; instead, it has implemented 'affordable housing' (*jingjishiyongfang*) and 'inexpensive rental housing' (*lianzufang*) policies. Simultaneously, residential mortgage lending began in 1998 to stimulate commodity housing consumption. Currently, three kinds of residential mortgages are used in China: individual account housing loans, which are funded by banks' consumer credit funds to individual households; authorized housing loans, which are granted by banks with the authorization of the public reserve fund management department using the public reserve deposit as the source of funding; and combined housing loans, which are funded by both public reserve deposits and banks' consumer credit funds.

Model 2:

$$capspace = \alpha 2 \cdot wage + \beta 2 \cdot afpolicy + \gamma 2 \cdot ilavc + C2 \qquad (4\text{-}2)$$

In that equation, $\alpha 2$, $\beta 2$ and $\gamma 2$ are coefficients, and $C2$ is constant term.

The housing consumption index, $capspace$, is the per-capita floor space of residential buildings. Affordable housing police index, $afpolicy$, is calculated as: [(floor space sold of affordable housing/floor space sold of residential buildings)× (average selling price of residential buildings/average selling price of affordable housing)], and it reflects monetarily benefited part of households' housing consumption through affordable housing policy. The public reserve fund system index, $ilavc$, is calculated as: [interest rate on authorized loan - interest rate on individual housing account loan], and it reflects how much consumers better off their housing services consumption through public reserve deposit system.

Using model 3, we can review the reasons for housing price appreciation. Housing prices change with the change of supply ($crtspa8$), change of building cost ($cr\cos t8$), and change of demand ($crpop8$), interest rate changes ($crione8$), and so on.

Model 3:

$$crprich8 = \alpha 3 \cdot crtspa8 + \beta 3 \cdot cr\cos t8 + \gamma 3 \cdot crpop8 + \eta 3 \cdot crione8 \qquad (4\text{-}3)$$

In that equation, $\alpha 3$, $\beta 3$, $\gamma 3$ and $\eta 3$ are coefficients.

The housing price change index, $crprich8$, is the rate of change of the average selling price of residential buildings (1998=100); the supply change index, $crtspa8$ is the rate of change of the total floor space of residential buildings (year-end) (1998=100). The building cost change index, $crcost8$ is the rate of change in the cost of buildings completed (1998=100). The demand change index, $crpop8$ is the rate of change in the urban population (1998=100). Finally, the interest rate change index, $crione8$ is the rate of change in the nominal legal interest rates on one-year loans (1998=100).

In model 4, the effects of purchasing power and urban amenities on housing consumption are examined. As Fig. 4-1 presents, consumer price general indices have changed little in contrast to income and housing prices. Therefore, the purchasing power index is represented by $wavpri$, the ratio of wage and housing price. Moreover, purchasing power much more contributes to housing consumption than urban amenities, and then we apply non-linear function form in model 4 taking logarithms on urban amenity indices.

Model 4:

$$\begin{aligned} capspace &= \alpha 4 \cdot wavpri + \beta 4 \cdot lperwater + \gamma 4 \cdot lpergas + \eta 4 \cdot lnpveh \\ &+ \delta 4 \cdot lcaproad + \varepsilon 4 \cdot lcapgreen + \mu 4 \cdot lnptoi + \nu 4 \cdot lcapprow \\ &+ \tau 4 \cdot lcap\lg pi + \rho 4 \cdot lcapslew + \theta 4 \cdot lcapdisw + C4 \end{aligned} \tag{4-4}$$

In that equation, $\alpha 4$, $\beta 4$, $\gamma 4$, $\eta 4$, $\delta 4$, $\varepsilon 4$, $\mu 4$, $\nu 4$, $\tau 4$, $\rho 4$, and $\theta 4$ are coefficients and $C4$ is the constant term.

Urban amenities indices are the following: $lperwater$ is the logarithm of percentage of population with access to tap water; $lpergas$ is the logarithm of percentage of population with access to gas; $lnpveh$ is the logarithm of the number of public transportation vehicles per 10,000 persons; $lcaproad$ is the logarithm of the per-capita area of paved roads; $lcapgreen$ is the logarithm of per-capita public green areas; $lnptoi$ is the logarithm of number of public lavatories per 10,000 persons; $lcapprow$ is the logarithm of the per-capita daily production capacity of tap water supply (year-end); $lcaplgpi$ is the logarithm of the per-capita length of gas pipe lines; $lcapslew$ is the logarithm of the per-capita length of city sewage; and $lcapdisw$ is the logarithm of the per-capita daily disposal capacity of city sewage.

Model 5 is used to gauge the effect of geographic inequity on housing consumption.

Utility function of proxy individual in a city is specialized as a Cobb-Douglas form with budget constraint.

$$U(x,q,G)=\alpha\ln x+\beta\ln q+\gamma\ln G \qquad (\alpha+\beta=1)$$

$$s.t. \quad y=x+p\cdot q$$

where x denotes numeraire composite goods, q is the housing consumption, G is urban amenity, p is housing price, and y is income level.

In the open cities, the utility level ($V(y_i)=\alpha\ln(\alpha y_i)+\beta\ln q_i+\gamma\ln G_i$) of households in city i equals the utility level ($V(y_j)=\alpha\ln(\alpha y_j)+\beta\ln q_j+\gamma\ln G_j$) of households in city j, in equilibrium. According to those relations, the difference of housing consumption between cities can be written as Eq. (4-5).

Model 5:

$$\begin{aligned} gcapspa &= \alpha5\cdot gwage+\beta5\cdot gpriceh+\gamma5\cdot g\cos t+\eta5\cdot gperwater+\delta5\cdot gpergas+\varepsilon5\cdot gnpveh \\ &+\mu5g\cdot gcaproad+v5\cdot gcapgreen+\omega5\cdot gnptoi+\rho5\cdot gcaplwpi \\ &+\theta5\cdot gcap\lg pi+\tau5\cdot gcaplhpi+\theta5\cdot gcaoslew+\lambda5\cdot gcapdisw+C5 \end{aligned} \tag{4-5}$$

In that equation, $\alpha5$, $\beta5$, $\gamma5$, $\eta5$, $\delta5$, $\varepsilon5$, $\mu5$, $v5$, $\omega5$, $\tau5$, $\rho5$, $\theta5$, and $\lambda5$ are coefficients and $C5$ is a constant term.

Model 5 is adjusted according to model 4, geographic inequity indices are represented by proportions of indices between the eastern area and the western, central areas. In that equation, $gcapspa$ is the proportion of housing consumption between an eastern area province and a western area province or between an eastern area province and a central area province; $gwage$ is the proportion of average annual money wage between an eastern area province and a western area province or between an eastern area province and a central area province; *gpriceh* is the proportion of housing price; *gcost* is the proportion of cost; *gperwater* is the proportion of percentage of population with access to tap water; *gpergas* is the proportion of percentage of population with access to gas; *gnpveh* is the proportion of number of public transportation vehicles per 10,000 persons; *gcapgreen* is the proportion of per-capita public green areas; *gcaproad* is the proportion of the per-capita area of paved roads; *gnptoi* is the proportion of number of public lavatories per 10,000 persons; *gcaplwpi* is the proportion of the per-capita length of water supply pipelines; *gcaplgpi* is the proportion of the per-capita length of gas pipelines; *gcaplhpi* proportion of the per-capita length of heating pipelines; *gcapslew* is the proportion of the per-capita length of city sewage; and *gcapdisw* is the proportion of the per-capita daily disposal capacity of city sewage.

4.3 Empirical results

We gauge our empirical model using OLS.

Model 1 examines the relationship between price and income, building cost, mortgage interest rate, and loan interest rates on developers. As Table 4-1 shows, construction costs contribute most to housing prices, and household income has a positive relation with housing prices. An increase of lending interest rates on developers' loans raises housing prices, whereas an increase of mortgage interest rates decreases the housing price because development costs increase with the lending interest rate increase, and a rising rate slows increases in the housing supply. Moreover, housing demand decreases with increasing mortgage interest rates. The elasticity of cost (0.883), lending interest rate on

Table 4-1 Model 1

Variable	Model 1
lwage	0.18841614***
lcost	0.88261948***
lilcom	0.35816246**
lilind	-0.66068***
R^2	0.999274

Legend: * p<0.1; ** p<0.05; *** p<0.01

developers' loans (0.358), and mortgage interest rate (0.661) are higher than the elasticity of wage (0.188), which implies that households' purchasing powers are feeble, and most of their housing consumption depends on housing loans.

Model 2 verifies the effectiveness of affordable housing policy. As Table 4-2 presents, residents have improved levels of housing service consumption more through affordable housing policy than through their increased income. However, the positive coefficient of the public reserve deposit system index is not significant. The result might be more interesting if data which indicate the proportion of authorized loan in mortgage market were applied.

Model 3 shows reasons for housing price appreciation. As Table 4-3 shows, the augmentation of construction costs, especially population growth, contributes much to the housing price rise because housing demand increases more rapidly than housing

Table 4-2 Model 2

Variable	Model 2
wage	0.00050308***
afpolicy	0.34670471***
ilavc	0.647904
_cons	16.685829***
R^2	0.727132

Legend: * p<0.1; ** p<0.05; *** p<0.01

Table 4-3 Model 3

Variable	Model 3
crtspa8	0.0279
crcost8	0.35578945***
crpop8	0.80354472***
crione8	-0.22561*
R^2	0.953706

Legend: * p<0.1; ** p<0.05; *** p<0.01

Model 3 shows reasons for housing price appreciation. As Table 4-3 shows, the augmentation of construction costs, especially population growth, contributes much to the housing price rise because housing demand increases more rapidly than housing supply in the urbanization process. It verifies the proceeding proposition that housing rent appreciates with population growth. Commonly, the legal lending interest rate has a positive effect on housing price appreciation, but we find a paradoxical result. The coefficient of *crione*8 is significantly negative, which might be interpreted as the reason that Chinese central bank has dropped interest rates of lending and deposit 8 times during 1996–2002 and raised them in 2004. For that reason, changes of interest rates negatively attribute to housing price appreciation during 1998–2005.

As Table 4-4 shows, the estimated relationship between housing consumption and urban amenities is desirable. Purchasing power, gas, public road, public green areas, and disposal capacity of city sewage are examined as positive and significant effects on housing consumption, although some of amenities such as tap water and public lavatories have significant negative effect conversely. A lot of public lavatories are independently constructed outside in Chinese cities, and they are nuisance to vicinity households, consequently, they negatively attribute to housing consumption. However,

Table 4-4 Model 4

Variable	Model 4
wavpri	0.52399564***
lperwater	-20.410705***
lpergas	18.495482***
lnpveh	1.746018
lcaproad	5.8292066***
lcapgreen	4.9515473**
lnptoi	-5.9008057***
lcaprow	-2.2927*
lcaplgpi	1.5382638**
lcaplsew	-0.52469
lcapdisw	1.1549566**
_cons	16.15408***
R^2	0.576773

Legend: * p<0.1; ** p<0.05; *** p<0.01

the negative effect of tap water on housing consumption is mysterious.

As Table 4-5 presents, housing consumption in different areas differs according to various factors. On housing consumption in the areas with geographic inequity, the most effective factor is cost, and amenities such as gas, public roads, and public green

Table 4-5 Model 5

Variable	Model 5
gwage	0.036788
gpriceh	-0.00478
gcost	0.23112693***
gperwater	0.081931
gpergas	-0.00602
gnpveh	-0.05777113**
gcaproad	0.13147702***
gcapgreen	0.12466437***
gnptoi	-0.05563*
gcaplwpi	-0.05064*
gcaplgpi	-0.00672
gcaplhpi	0.000773
gcaplsew	0.003823
gcapdisw	-0.00045
_cons	0.65662669***
R^2	0.60674

Legend: * p<0.1; ** p<0.05; *** p<0.01

areas play a very important role in determining housing consumption. In addition, nuisance amenities such as public transportation vehicles and public lavatories negatively contribute to housing consumption because of noisy, congestion, stench, and so on. However, income inequities are not attributable to housing consumption. It might be interpreted as the reason that high income is cancelled out by relatively high housing price in developed cities, although the collinearly does not exit in this model.

5 Concluding Remarks

This paper developed a dynamic general equilibrium model for housing market and analyzed the performance of the housing market in Chinese cities since the tenure-housing policy was implemented.

We interpreted the reason for the implementation of affordable housing policy and its effective implementation. Under an immature housing market, "housing construction" policy is the sole means of improving the residential circumstances for low-income households. Land rent in an upscale residential area is lower than that of a lower-scale residential area; moreover, subsidy policy to developers play a more effective role when they induce lower-income households to be located in higher land-rent areas to consume more housing services. In addition, the operating period of affordable housing lengthens with either higher construction quality, higher land rent, a higher rate of capital return, or a lower discount rate.

Empirical analysis was used to analyze the status of Chinese urban housing market. The households' purchasing powers are feeble and most of their housing consumption depends on mortgages. Under this situation, affordable housing policy attributes much more to the improvement of residential circumstances than increased income. Population growth contributes much more to the housing price appreciation than increased construction cost. Some urban amenities such as gas, public road and public green areas have significant positive effect on housing consumption, while nuisance amenity like public lavatories has significant negative effect.

This paper has analyzed the performance of the Chinese urban housing market, but population growth and public goods service changes were not considered. Extension of the model to incorporate those problems remains as a problem for future research.

Appendix

1. Proof that the housing operating period equals the land-lease period ($S = T$)

The profit of commodity housing per unit of land area is defined as Eq. (A-1-1).

$$\pi = b(d)\int_0^S [R(q(t)) - m(t)]e^{-rt}dt - e^{-ra}\int_T^S l(a,d)dt - L(T,d) - C(b,q0) \qquad \text{(A -1-1)}$$

$$\text{s.t. } \dot{q} = f(m,q) \qquad \text{(A-1-2)}$$

$$m(t) >= 0 \qquad \text{(A-1-3)}$$

$$q(t) > 0 \qquad \text{(A-1-4)}$$

where $a = Min[T,S]$.

Developers operate commodity housing to maximize Eq. (A-1-1): their profits. The operating income is described as the first term of the right-hand-side in Eq. (A-1-1).

Its second term is the extra lump-sum land rent (residual property income from rest years of land use rights) that must be paid for (S-T) period when S>T (S<T). Moreover, it is calculated using the earlier land rent because of its lump-sum payment mode.

The current value Hamiltonians for Eq. (A-1-1) with respect to m is

$$H = b(d)[R(q(t)) - m(t)] + \lambda(t) f(q,m), \qquad \text{(A-1-5)}$$

where $\lambda(t)$ is the adjoint variable.

State equation:

$$H_{\lambda} = \frac{\partial H}{\partial \lambda} = f(q,m) \qquad \text{(A-1-6)}$$

Multiplier equation:

$$\dot{\lambda}(t) = \frac{\partial \lambda}{\partial t} = r\lambda - \frac{\partial H}{\partial q} = r\lambda - b(d)R_q - \lambda f_q \qquad \text{(A-1-7)}$$

Optimality condition:

$$H_m = \frac{\partial H}{\partial m} = -b(d) + \lambda f_m = 0 \qquad \text{(A-1-8)}$$

From the boundary and transversality conditions, we derive the following relations.

$$\lambda(0) = \frac{\partial C(b,q0)}{\partial q0} \qquad \text{(A-1-9)}$$

$$H(S) = \begin{cases} [r(T-S)+1]l(S,d), & when \quad T > S \\ e^{rT}\bar{l}, & when \quad T = S \\ e^{r(S-T)}l(T,d), & when \quad T < S \end{cases} \qquad \text{(A-1-10)}$$

The maximized profit functions of commodity housing can be rewritten using the preceding equations as the following.

$$
\begin{aligned}
\pi^* &= b(d)\int_0^S [R(q(t)) - m(t)]e^{-rt}dt - e^{-ra}\int_T^S l(a,d)dt - L(T,d) - C(b,q0) \\
&= \int_0^S b(d)[R(q(t)) - m(t)]e^{-rt}dt - e^{-ra}\int_T^S l(a,d)dt - L(T,d) - C(b,q0) \\
&= \int_0^S (H - \lambda f)e^{-rt}dt - e^{-ra}\int_T^S l(a,d)dt - L(T,d) - C(b,q0) \\
&= \frac{1}{r}[H(0) - e^{-rS}H(S)] - e^{-ra}\int_T^S l(a,d)dt - L(T,d) - C(b,q0)
\end{aligned}
$$

The following can then expressed.

$$
\pi^* = \begin{cases}
\frac{1}{r}H(0) - \frac{e^{-rS}}{r}l(S,d) - L(T,d) - C(b,q0), & when \quad T > S \\
\frac{1}{r}H(0) - L(T,d) - C(b,q0) - \frac{\bar{l}}{r}, & when \quad T = S \\
\frac{1}{r}H(0) - e^{-rT}(S - T + \frac{1}{r})l(T,d) - L(T,d) - C(b,q0), & when \quad T < S
\end{cases} \qquad \text{(A-1-11)}
$$

If assumption $e^{-rS}l(S,d) = e^{-rT}l(T,d) = \bar{l}$ is applied, the profit of commodity housing is the maximum in Eq. (A-1-11), when developers operate commodity housing for either T period or $S(T > S)$ period.

However, developers would not like to repeat the construction-demolition cycle frequently in reality because of demolition costs and housing durability. For that reason, the operating period of housing is determined as Eq. (A-1-12) in this paper.

$$
S = T \qquad \text{(A-1-12)}
$$

References

Arnott, R.J., Davidson, R., Pines, D., 1983. Housing quality, maintenance and rehabilitation. *Review of Economics Studies* 50, 467–494.

Arnott, R.J., Braid, M.R., 1997. A filtering model with steady-state housing. *Regional Science and Urban Economics* 27, 515–546.

Braid, R.M., 1986. The comparative statics of a filtering model of housing with two income groups. *Regional Science and Urban Economics* 16, 437–448.

Brueckner, J.K., 1981. A dynamic model of housing production. *Journal of Urban Economics* 10, 1–14.

Deng, F.F., 2005. Public land leasing and the changing roles of local government in urban China. *The Annals of Regional Science* 39, 353–373.

Deng, Y., Zheng, D., Ling, C.G., 2004. An early assessment of residential mortgage performance in China. *Journal of Real Estate Finance and Economics*, 31(2), 117-136.

Frame, D.E., 2004. Equilibrium and migration in dynamic models of housing market. *Journal of Urban Economics* 55, 93–112.

Fujita, M., 1982. Spatial patterns of residential development. *Journal of Urban Economics* 12, 22–52.

Glaeser, E.L., Gyourko, J., Saks, R.E., 2005. Urban growth and housing supply. *National Bureau of Economic Research*, NBER Working Paper, No. 11097.

Goodman, A.C., 2003, Following a panel of stayers: length of stay, tenure choice, and housing demand. *Journal of Housing Economics* 12, 106–133.

Hardman, A.M., Ioannides, Y.M., 1999. Residential mobility and the housing market in a two-sector neoclassical growth model. *Scandinavian Journal of Economics* 101, 315–335.

Jiang, X.M., Sasaki, K., 2006. The effects of tenure-housing policy on urban spatial structure and social welfare in China. Mimeo.

Miceli, T.J., Sirmans, C.F., 2005. Time-limited property rights and investment incentives. *The Journal of Real Estate Finance and Economics* 31, 405–412

Ohls, J.C., 1974. Public policy toward low income housing and filtering in housing markets. *Journal of Urban Economics* 2, 144–171.

Roger C.K. Chan., Y.M. Yao., Simon X.B. Zhao., 2003. Self-help housing strategy for temporary population in Guangzhou, China. *Habitat International* 27, 19–35.

Sasaki, K., 1990. An alternative version of a vintage model of an urban housing market. *Journal of Urban Economics* 28, 125–139.

Sasaki, K., 1987. A comparative static analysis of urban structure in the setting of endogenous income. *Journal of Urban Economics* 22, 53–72.

Siqi Zheng.,Yuming Fu., Hongyu Liu., 2006. Housing-choice hindrances and urban spatial structure: Evidence from matched location and location-preference data in Chinese cities. *Journal of Urban Economics* 60, 535–557. *Habitat International* 31, 177–192.

Stephen W.K. Mark., Lennon H.T. Choy., Winky K.O. Ho., 2007. Privatization, housing conditions and affordability in the People's Republic of China

Sweeny, J.L., 1974. A commodity hierarchy model of the rental housing market. *Journal of Urban Economics* 1, 288–323.

Thijs ten Raa., Haoran Pan., 2005. Competitive pressures on China: Income inequality and migration. *Regional Science and Urban Economics* 35, 671–699.

Turnbull, G.K., 1988. Market structure, location rents, and the land development process. *Journal of Urban Economics* 23, 261-277.

Zheng, S., Fu, Y., Liu, H., 2006. Housing-choice hindrances and urban spatial structure:

Evidence from matched location and location-preference data in Chinese cities. *Journal of Urban Economics* 60, 535–557.

Zhou Yu., 2006. Heterogeneity and dynamics in China's emerging urban housing market: two sides of a success story from 1990s. *Habitat International* 30, 277–304.