Dynamic control of rural-urban migration

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Abstract

This study is an investigation of the optimal process of urbanization with a two-sector model comprising a rural and an urban sector. Not only long-run steady states, but also the processes of convergence to them are optimally controlled with a positive time discount in the present analysis. Results of the study are two-fold: i) Even if an urbanized steady state with higher output is possible, to realize it is not desirable if the funds for development are supplied at a high interest rate; ii) If an economy approaches an urbanized steady state from a non-urbanized steady state via a take-off, a government should accelerate rural-urban migration in early stages of development while slowing and restricting it in later stages.

Key words: *optimal urbanization control, big push, speed of urbanization*

JEL classification: O22, R11

1 Introduction

According to World Urbanization Prospects [14], the urbanization rate for the entire world increased greatly from 29% to 47% during the latter half of the twentieth century.

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Although urbanization in more-developed countries began to converge during those years, less-developed countries remain in the midst of rapid urbanization [1].

The purpose of this study is to investigate the optimal process of rural-urban migration along with economic development from theoretical and dynamic points of view. Completion of the entire process of urbanization takes much time. For that reason, we cannot remain unconcerned about the social benefits of the urbanization process and the long-run steady state realized as a result of urbanization. Such an issue is especially important when considering a less-developed economy in the middle of or before rapid urbanization. In each stage of development, what kind of intervention into laissez-faire urbanization process is required? To answer that question, this study investigates the optimization problem of inter-sectoral population distribution in a simple two-sector dynamic model.

Economic development with industrialization is a process of urbanization. The technology of production is labor intensive and land intensive, with constant returns to scale in the primitive economy. Subsequently, it becomes capital intensive in the modern economy, with increasing returns to scale resulting from positive externalities. In such a process of industrialization, people move from rural to urban areas, where many factories and firms with modern technology are concentrated. Such urbanization processes have been described in labor-surplus models e.g. Lewis [7] and Fei and Ranis [2]. Such a cluster of various economic activities in cities with definite places is well known to yield not only negative externalities by congestion, but also higher productivity through so-called agglomeration economies. As a result, the net benefit of a city is the non-monotonic increase in its size. A typical and early example of such a study is the system of cities model presented by Henderson [3]. Furthermore, the dynamic rural-urban model

[1] According to the data tables included in World Urbanization Prospects [14], the urbanization rate of more-developed countries has grown by only 0.9% during 2000–2005, but by 2.6% in less-developed countries during the same period.

of Bertinelli and Black [1] is an achievement in recent years, which investigated growth of the urban sector [2].

This study develops the rural-urban model of Bertinelli and Black [1] by considering dynamic optimization of urbanization processes. The economy of the presented model comprises two sectors: a rural sector with constant returns to scale, and an urban sector with both positive and negative externalities. Although the basic structure resembles that of the model of Bertinelli and Black [1], the present study drastically simplified their original model. Endogenous accumulation of human capital was considered in Bertinelli and Black [1], although the present study uses neither capital nor human capital: only the population of the urban sector in each period is a matter for consideration. This study might be criticized for such an omission of the model. However, this study was undertaken to solve the dynamic optimization problem by government, which was not considered in Bertinelli and Black [1]; moreover, the model must be simplified to treat the problem analytically.

In the present model, the positive externality is not a simultaneous effect but rather a dynamic effect with which the current scale of the urban sector influences future productivity. This assumption expresses that various economic activities accumulated in the past play important roles in production in the urban sector. A connection of a commercial transaction and a mutual trust between firms, along with a know-how in procedure of business deal in that regard, for example, have been established via past transactions in the cities. If we consider the growth of cities, such a dynamic externality is important as well as the simultaneous agglomeration economies, e.g. the range of variety [3]. On the

[2] Agglomeration economies in the urban sector are a kind of *demand-pull* force of urbanization. However, *supply-push* force that is attributable to a rapid increase in population in the rural sector also drives urbanization. Although the poverty of the pushed population has been recognized as a serious problem in recent years, this study only considers the *demand-pull* force to emphasize the aspect that urbanization contributes to sound industrialization and economic growth.

[3] The idea that the current stock of (human) capital influences the accumulation of new (human) capital was presented in the endogenous growth models of Lucas [8] and Romer [12]. A similar setting was also adopted in Bertinelli and Black [1]. In the present study, their idea is represented in a simple form

other hand, the congestion externality described in this study is a simultaneous effect, as in previous models.

Under the combination of dynamic positive externality and static negative externality, the optimal size of the urban sector is determined in the trade-off between present congestion and future growth. This study subsumes a lesser-developed country that is not urbanized in the initial period. The central government is supplied an unbounded but costly fund by ODA from developed countries; the government fosters urbanization with that fund to maximize dynamic social welfare. The interest rate of ODA serves as a discount factor of time to represent costs of funds in that optimization problem.

The analyses described in this paper are two-fold. First, this study evaluates whether the big push is truly desirable given a positive discount factor. In models with increasing returns to scale, multiple stable equilibria are well known to appear frequently. For that reason, once it is trapped into an underdeveloped equilibrium, the coordination of economic activities by the authority, a so-called big push, is necessary to lead the economy to the developed equilibrium. The necessity of big push was supported analytically by Murphy et al. [11] for the first time, and by succeeding analyses of Matsuyama [9], Iwaisako [4], and Murata [10]. The models of Bertinelli and Black [1] and the present study also include multiple equilibria with lower and higher rates of urbanization. However, this study does not simply support big push even if the economy enjoys a higher level of output in the urbanized steady state. We evaluate the process of convergence to the new equilibrium under a positive discount factor or interest rate of ODA [4]. Results of our analysis show that the big push is desirable only when ODA is funded at an appropriately low interest rate.

Second, from a dynamic point of view, this study reconsiders the well known result

[4]Although the importance of cost and benefit analysis for big push is addressed by the two period model of [11], we develop the analysis in the model without the terminal period and with ODA from abroad.

that cities are overpopulated. Henderson [3] derived a result that cities under laissez-faire are always larger than the optimal size under the assumption that the benefit of each city is single-peaked in its size. That result has been supported by results of various theoretical and empirical studies of Kanemoto [5] and Kanemoto et al. [6] [5]. However, their results mainly addressed the equilibrium sizes of cities; the present study examines growing cities. The process of urbanization, especially its rapidity, is addressed in our analysis. Results indicate that growth of the urban sector has priority; urbanization is accelerated in the initial stages of urbanization, but urbanization should be slowed down in the latter stage of urbanization to restrict excess congestion and provide a soft landing on the optimal steady state.

The present paper is organized as follows: Section 2 presents the model and its behaviors under laissez-faire. Section 3 describes formulation of the dynamic optimization problem by the government and solves the problem. Section 4 describes salient conclusions of this study.

2 The model and a laissez-faire economy

2.1 Model

This study considers a small open economy that consists of rural and urban sectors. The economy is closed with population, there are constant $\bar{N}$ numbers of homogeneous population in each period. Each person survives for only one period, during which time the worker works and consumes. At the beginning of each period, individuals can choose sectors to engage in without cost and thereby maximize their respective utility levels.

The two sectors input only labor to produce homogeneous output. They differ only in their technologies [6]. In addition, this study subsumes that each sector includes only

[5]Their conclusions are not necessarily preserved in a model that consists of only a few cities. Papageorgiou and Pines [13]. They investigated a two-region model in which both positive and negative externalities pertain within each region. Those studies show examples that the concentration in the larger city can be either too large or too small.

[6]We assume that the two sectors produce different outputs in their factories, but that they are sold

one-worker firms.

The rural sector produces output under constant returns to scale. Therefore, the productivity of each worker in the unit of the money is denoted by $g \geq 0$. Consequently, the consumable income of a worker in the rural sector is always g.

The technology of the urban sector is that of increasing returns to scale for the inter-temporal externality of production. The present productivity of each worker in the urban sector increased during the previous period; per-worker productivity in period t is denoted as n_{t-1}^{δ} with $0 < \delta < 1$, where n_s represents the population of the urban sector in period s. On the other hand, a negative externality from congestion increases in the current size of the sector. The negative externality is displayed as a monetary cost for living for each urban sector resident, which is denoted as bn_t. We denote the consumable income of a worker in the urban sector in period t as $y(n_{t-1}, n_t) = n_{t-1}^{\delta} - bn_t$.

2.2 Urbanization under laissez-faire

In this model, households choose the sector to maximize their static consumable income because they survive for one period only. Therefore, $y_t(n_{t-1}, n_t) = g$ must hold if a $n_t \in (0, \bar{N})$ is in static equilibrium for a given n_{t-1}. Furthermore, $y_t(n_{t-1}, 0) \leq g$ and $y_t(n_{t-1}, \bar{N}) \geq g$ must hold respectively for $n_t = 0$ and $n_t = \bar{N}$ to be in equilibrium. The current urban scale brings only negative externalities. For that reason, the utility of the urban sector decreases in its current size. Consequently, n_t is determined uniquely for given n_{t-1}. As a result, the static equilibrium of location of households is represented as a function $n_t = Q(n_{t-1})$, which is shown as follows.

$$Q(n_{t-1}) = \begin{cases} 0 & \text{if} \quad 0 \leq n_{t-1}^{\delta} \leq g \\ (n_{t-1}^{\delta} - g)/b & \text{if} \quad g < n_{t-1}^{\delta} < g + b\bar{N} \\ \bar{N} & \text{if} \quad g + b\bar{N} \leq n_{t-1}^{\delta} \leq \bar{N} \end{cases} \tag{1}$$

in a global market and changed into homogeneous consumption goods or money. Therefore, the two sectors are apparently producing the same type of good directly. For simplicity, it is assumed that the prices of the goods are given and fixed.

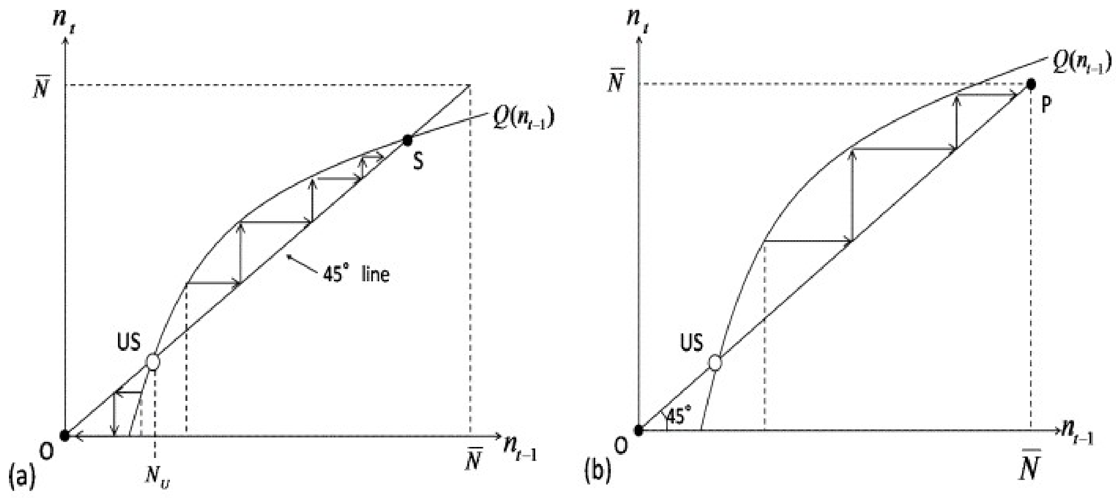

Figure 1 Urbanization under laissez-faire

The process of economic development of this model is shown as the sequence of these static equilibria. The phase diagram is given in Figs. 1(a) and 1(b), in each of which $n_t = Q(n_{t-1})$ and a 45 line are drawn.

In Fig. 1(a), $n_t = Q(n_{t-1})$ and $n_t = n_{t-1}$ are shown to have two intersections: US and S. In that figure, two stable steady states, O and S, and an unstable one, US, exist; the process of convergence and which equilibria to be realized depend on the initial size of the urban sector, which is given as n_0. If $n_0 > N_U$ is given, the economy converges monotonously to S, but converges to O if $n_0 < N_U$; therefore, N_U is the threshold level of urbanization. Regarding the convergence process which approaches the origin, the urban sector shrinks and industrialization does not occur.

The state in which the economy is trapped into the less urbanized equilibrium is designated as a development trap in Bertinelli and Black [1]. To escape from the trap, they also support that the government should lead the economy to the converging path to the more-industrialized equilibrium, N^*, by attracting labor to the urban sector with subsidies or some other inducement. Generally, such coordination by the authority is called a big push.

Figure 1(b), in which $n_t = Q(n_{t-1})$ and $n_t = n_{t-1}$, shows only one intersection – US; consequently, it has no interior equilibrium. Instead, $n_t = \bar{N}$, the state in which the entire population lives in urban areas, is the long-run equilibrium.

3 Optimal control of urbanization

This section presents analyses of the optimal urbanization process from a dynamic point of view. In each stage of economic development, what kind of intervention of government is desirable? To determine the optimal sequence of policy to advance economic development, this study addresses a government of the small open country; its optimal policy is derived as a maximization of its dynamic objective function.

3.1 Optimization problem

It is assumed that the government can borrow funds freely with a constant interest rate $r \geq 0$, which is funded by development assistance from more-developed countries. Furthermore, it is assumed that the government freely redistributes the income among sectors through taxation and subsidies. The subsidy to each household in the urban (rural) sector in period t is denoted as $k_{ut}(k_{rt})$; taxes are represented by their negative value. Because the government can borrow money without an upper bound, taxes and subsidies need not be balanced within a period; only $\Sigma_{t=1}^{\infty}\rho^{t-1}[k_u n_t + k_r(\bar{N} - n_t)] = 0$ must hold.

With such a redistribution of income, the government can intervene in the population distribution between sectors. Because not k_{ut} and k_{rt}, but only their difference, denoted as $k_{ut} - k_{rt}$, affects the population distribution, only $k_{ut} - k_{rt}$ is related to control of the population distribution. The word "subsidy" is used hereinafter in relative terms to concentrate on the influence to population distribution. The urban sector is said to be subsidized (taxed) in period t if $k_{ut} - k_{rt}$ is positive (negative).

If the ultimate objective of the government is to maximize a dynamic social welfare, two steps can achieve such optimization: maximize the current value of aggregate output through inter-sectoral redistribution; redistribute it between generations. The government controls two variables k_{ut} and k_{rt} in this model. Therefore, the first and second steps are treated as independent problems. Once the aggregate output is maximized, an appropriate redistribution of it necessarily achieves social optimum irrespective of the setting of the social welfare function [7]. Therefore, this study merely highlights the first problem, or output maximization, because we do not need a two-sector model like this to treat the second one.

Although the government indirectly controls the urban population n_t via $k_{ut} - k_{rt}$ in the first setting of the model, the problem is described as if n_t is directly controlled for simplicity of description. The problem that the government faces in the initial period $t = 1$ is described as [8]

$$\max_{n_1, n_2, \cdots} \Sigma_{t=1}^{\infty} (n_{t-1}^{\delta} n_t - g n_t - b n_t^2) \rho^{t-1}, \tag{2}$$

where n_0, the initial size of the urban sector, is given in this problem. Also, $\rho \equiv 1/(1+r)$ is the time discount factor which holds $0 \leq \rho \leq 1$.

The first-order condition or Euler equation of the problem described above is shown

[7] $\Sigma_{t=1}^{\infty} \rho^{t-1} [k_u n_t + k_r(\bar{N} - n_t)] = 0$ must hold for budget constraint of the government. Therefore, if we respectively denote the total output and consumption in period t as Y_t and C_t, then $C_t = Y_t + (k_{ut} - k_{rt}) n_t + k_{rt} \bar{N}$ and $\Sigma_{t=1}^{\infty} \rho^{t-1} C_t = \Sigma_{t=1}^{\infty} \rho^{t-1} Y_t$ hold. Even if the government maximizes $\Sigma_{t=1}^{\infty} \rho^{t-1} Y_t$ by determining $k_{ut} - k_{rt}$ in each period, any C_t is feasible as long as $\Sigma_{t=1}^{\infty} \rho^{t-1} C_t = \Sigma_{t=1}^{\infty} \rho^{t-1} Y_t$ holds because k_{rt} is independent of $k_{ut} - k_{rt}$ if k_{ut} is controlled appropriately. Therefore, to maximize $\Sigma_{t=1}^{\infty} \rho^{t-1} Y_t$ is independent of a problem by which a social welfare described by $W(C_1, \cdots)$ is maximized with respect to $C_1, \cdots$.

[8] Equation (2) maximizes the aggregate profit of the urban sector instead of GDP, but those two problems are equivalent because $\max_{n_1, n_2, \cdots} \Sigma_{t=1}^{\infty} [n_{t-1}^{\delta} n_t - b n_t^2 + g(\bar{N} - n_t)] \rho^{t-1} = \max_{n_1, n_2, \cdots} \Sigma_{t=1}^{\infty} (n_{t-1}^{\delta} n_t - g n_t - b n_t^2) \rho^{t-1} + \Sigma_{t=1}^{\infty} \rho^{t-1} g \bar{N}$ holds.

as

For all $t \geq 1$,

$$n_{t-1}^{\delta} - 2bn_t - g + \rho\delta n_{t+1} n_t^{\delta-1} = 0 \quad \text{if} \quad 0 < n_t < \bar{N} \tag{3}$$

$$n_{t-1}^{\delta} - 2bn_t - g + \rho\delta n_{t+1} n_t^{\delta-1} \leq 0 \quad \text{if} \quad n_t = 0 \tag{4}$$

$$n_{t-1}^{\delta} - 2bn_t - g + \rho\delta n_{t+1} n_t^{\delta-1} \geq 0 \quad \text{if} \quad n_t = \bar{N}. \tag{5}$$

However, the first-order condition is insufficient to identify the unique optimal path. We derive another necessary condition for the optimal path to exclude the surviving paths from the first-order conditions. If a path is optimal, following Lemma 1 must be true for all points on that path:

Lemma 1

On the optimal path, the following hold: i) if $n_t > n_{t-1}$, $n_{t+1} \geq n_t$ holds, ii) if $n_t < n_{t-1}$, $n_{t+1} \leq n_t$ holds, iii) if $n_t = n_{t-1}$, $n_{t+1} = n_t$ holds.

The proof is noted in Appendix 1. That is, n_t must be either monotonously increasing or decreasing in t on the optimal path. Lemma 1 might be readily apparent because it excludes inefficient paths, which do not converge directly to those destinations.

From the first-order conditions (3) to (5) and Lemma 1, we can derive a transition equation [9] as the following.

$$n_{t+1} - n_t = \begin{cases} n_t^{1-\delta}(2bn_t - \rho\delta n_t^{\delta} - n_{t-1}^{\delta} + g)/(\rho\delta) & \text{if} \quad 0 < n_t < \bar{N} \\ 0 & \text{if} \quad n_t = 0, \bar{N} \end{cases} \tag{6}$$

[9] The transition in $0 < n_t < \bar{N}$ is a simple reform of (3). If $n_t = \bar{N}$ holds, $n_t \geq n_{t-1}$; hence $n_{t+1} = \bar{N}$ holds because of Lemma 1. Furthermore, if $n_t = 0$ holds, $n_t \leq n_{t-1}$; hence $n_{t+1} = 0$ holds because of Lemma 1. Therefore, (6) is derived.

3.2 Phase diagrams

In this subsection, phase diagrams are drawn to analyze the optimal urbanization process. The phase diagram is drawn in (n_{t-1}, n_t) coordinates. To describe the changes of the variables, $\Delta n_{t-1} \equiv n_t - n_{t-1}$ and $\Delta n_t \equiv n_{t+1} - n_t$ are defined.

First, reform of (6) yields the loci on which $\Delta n_t = 0$ as the following equation.

$$n_{t-1} = (2bn_t - \rho\delta n_t^{\delta} + g)^{1/\delta} \tag{7}$$

$$n_t = 0 \tag{8}$$

$$n_t = \bar{N} \tag{9}$$

The locus expressed by (7) is drawn by a convex curve. From (6), Δn_t is positive (negative) on the left (right) of that locus.

Second, the $\Delta n_{t-1} = 0$ locus is drawn as the 45 line crossing the origin in (n_{t-1}, n_t) coordinates because $\Delta n_{t-1} = 0$ is equivalent to $n_t = n_{t-1}$. Δn_{t-1} is positive (negative) on the left (right) of the locus.

Phase diagrams are shown as Figs. 2(a)–2(c), with which we derive the optimal urbanization path. Figures 2(a) and 2(b) differ only in the horizon of total population $\bar{N}$; others are the same. In Fig. 2(a), where the total population is sufficiently large, the internal steady state or the intersection of $\Delta n_{t-1} = 0$ and $\Delta n_t = 0$ loci is constrained by total population, but is crowded out of the bounds in Fig. 2(b), in which the total population is small. Furthermore, Fig. 2(c) depicts a situation in which $\Delta n_{t-1} = 0$ never intersects $\Delta n_t = 0$, unlike Figs. 2(a) and 2(b).

In these diagrams, n_{t-1} is the state variable and n_t is the jumping variable or policy variable controlled by the government. In period t, n_{t-1} has already been determined from the behavior in the previous period and is therefore given, although n_t is freely controlled to maximize the objective function. However, n_t becomes the state variable in the next period.

Before investigating the behaviors of those diagrams, we derive the conditions of parameters by which each case arises. First, substituting $n_{t-1} = n_t = n$ into (7) yields

$$(1 + \rho\delta)n^{\delta} - 2bn = g. \tag{10}$$

The intersections of $\Delta n_{t-1} = 0$ and $\Delta n_t = 0$ loci are derived if we solve (10) with n, and N_E is represented by their larger solution. The case of Fig. 2(a) or 2(b) arises when (10) has real solutions. Its condition is described as

$$g \leq (1 - \delta)(1 + \rho\delta)^{1/(1-\delta)}(\delta/2b)^{\delta/(1-\delta)}. \tag{11}$$

That condition holds if g and b are small. Unless the condition described above holds, the case of Fig. 2(c) arises. In addition, Fig. 2(a) arises when N_E is less than $\bar{N}$. For $N_E < \bar{N}$ to be satisfied, parameters must meet the following conditions:

$$(1 + \rho\delta)\bar{N}^{\delta} - 2b\bar{N} < g \tag{12}$$

$$\delta(1 + \rho\delta)\bar{N}^{\delta-1} - 2b < 0. \tag{13}$$

Intuitively, the conditions described above hold if and only if $\bar{N}$ is sufficiently large. Figure 2(b) arises if $\bar{N}$ is small and the conditions do not hold.

We investigate the behavior of Fig. 2(a) first. Four types of path exist in this phase diagram: (i) the paths converging to P, (ii) the paths swerving around D, (iii) those converging to O, (iv) the unique saddle path to the internal steady state E. Among those, two specific paths remain but others are excluded by necessary conditions.

First, if $n_{t-1} = n_t = \bar{N}$ holds; $n_{t+1} > n_t = \bar{N}$ must hold for the first-order condition (5) to be satisfied because point P is above the $\Delta n_t = 0$ locus represented as (7). Therefore, the first-order condition is not satisfied in P. For that reason, all paths converging to that point are excluded. Second, the paths swerving around D are excluded by Lemma 1. Third, if we neglect the discreteness of the model and assume the behavior of the diagram to be continuous, all the paths to the origin are excluded aside from that which

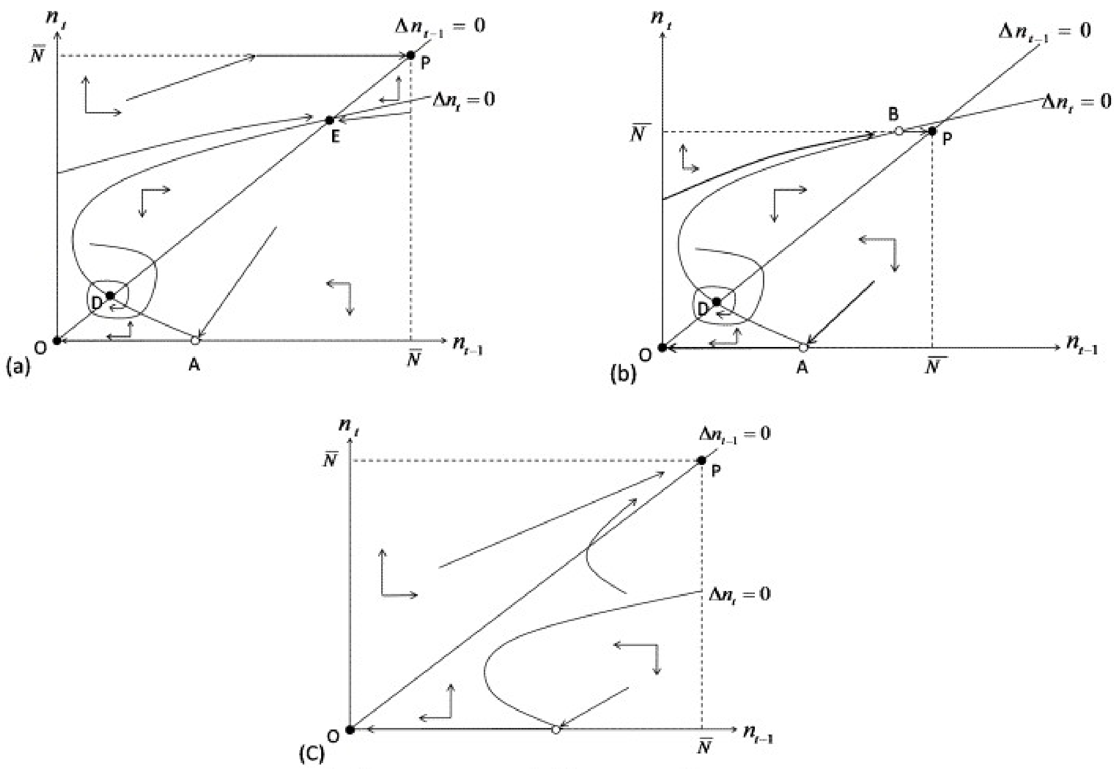

Figure 2 Phase diagrams

meets $n_t = 0$ at Point A because they do not hold the first-order condition when they meet $n_t = 0$ in the right of A [10]. The remaining path also satisfies Lemma 1 because it is monotonic. Consequently, that path is not excluded by any necessary conditions. Fourth, the saddle path to E holds the first-order condition (3) and Lemma 1; hence it also survives. We denote the coordinates of E as (N_E, N_E).

Conclusively, two paths remain in Fig. 2(a) which cannot be rejected by any necessary conditions: one is the path converging to O via A; another is the saddle path to E. We must compare them to determine the optimal one among those, but the answer for that question is left until the next subsection.

We next describe the case of Fig. 2(b) in which the internal steady state E is crowded out the binding of total population. Consequently, three types of paths exist. First, also

[10]The conclusion is slightly different when we strictly consider the discreteness of the model. If the behaviors are diagrammed discretely, paths can leap the locus of $\Delta n_t = 0$; they might strike $n_t = 0$ in the left of A. Such paths hold the first-order condition because (4) is satisfied on segment OA, which is left of $\Delta n_t = 0$. It is not surprising that all such paths are optimal, although thousands of them exist. Each path passes a finite number of points in the discreteness of the model. Consequently, the optimal path for each initial state is not expressed by a unique curve, unlike the continuous model.

in that case, the path via A survives among the paths converging to the origin. Second, the paths swerving around D are excluded by Lemma 1. Third, some paths are shown to converge to P in this figure. Among those paths, that which intersects $n_t = \bar{N}$ at B is optimal if we regard the behavior of the diagram as smooth. In that path, the first-order condition is satisfied when the economy arrives at the steady state. The other paths which strike $n_t = \bar{N}$ in the left of point B are excluded because they do not hold the first-order condition (5) such as paths converging to P in Fig. 2(a) [11] Therefore, the two paths survive also in Fig. 2(b).

The last is represented in Fig. 2(c), where $\Delta n_{t-1} = 0$ and $\Delta n_t = 0$ loci never mutually intersect. In that figure, all paths converging to P are excluded from first-order conditions (5) as in Fig. 1(a); only the path converging to O via A survives among the first-order conditions. Therefore, without the analysis in the next subsection, the urban sector is shown to diminish in the optimal steady state in that case.

3.3 Selection of the optimal path

In the previous subsection, two candidates of the optimal paths exist in Figs. 2(a) and 2(b): paths converging to the steady state E or P, say urbanized steady states, and that to O, called the non-urbanized steady state. In the present subsection, we investigate which is optimal by comparing them. Although the present analysis specifically examines paths with urbanization, the efficiency of internal urbanized steady states is discussed before choosing the optimal path. Throughout the analysis of this subsection, we only highlight the case represented in Fig. 2(a), where internal steady states exist. Therefore, it is presumed that (10) holds to exclude Fig. 2(c) and that $\bar{N}$ is appropriately large to exclude Fig. 2(b).

[11] All the paths which strike $n_t = \bar{N}$ in the right of B are optimal when we strictly consider discreteness of the model.

3.3.1 Efficiency of the urbanized steady state

First, we investigate when the urbanized steady state is efficient. A steady state $n_{t-1} = n_t = N_E$ is called efficient if $N_E^{\delta} - bN_E > g$ is satisfied in a steady state and the per-period net output of one worker of the urban sector is less than that of the rural sector. Otherwise, the steady state is called inefficient. Because we neglect the constraint of total population by assuming that $\bar{N}$ is appropriately large for simplicity of analysis, N_E is an internal steady state.

Urbanization is never a desirable outcome if the internal steady state is inefficient, or if the steady state is inferior to the non-urbanized steady state represented by O in Figs. 2(a) and 2(b). For an efficient internal steady state, the parameters must hold the following conditions represented by Lemma 2.

Lemma 2

An efficient internal steady state exists if the following hold: i) *condition (11) holds when* $\rho \leq 1/(2-\delta)$, ii) $g > (1-\rho\delta)(\rho\delta/b)^{\delta/(1-\delta)}$ *holds when* $\rho > 1/(2-\delta)$.

The detailed process to derive Lemma 2 is presented in Appendix 2. That Lemma states that no inefficient urbanized steady state can be excluded by the necessary conditions. There might be an inefficient urbanized steady state in the phase diagram if ρ, b, and g are appropriately high.

3.3.2 Desirability of a big push

The paths with urbanization are never selected as the optimal path if an efficient urbanized steady state is not possible. However, the paths to the efficient urbanized steady state are not necessarily optimal when we evaluate them using a positive time-discount value. In the present inquiry, we therefore investigate whether urbanization is desirable or not. The set of parameters is assumed to hold the conditions in Lemma 2 to promise the existence of an efficient internal solution. In addition, $n_0 = 0$ is given as the initial state, although the evaluations of converging paths are dependent on the initial state. Such an initial state is an equilibrium under laissez-faire, which is caught in a development trap. For that reason, a big push is necessary so that the economy leaves it for urbanized states. Considering such an initial situation, we investigate the question of whether an undeveloped economy should be developed by a big push.

If we determine the optimal path explicitly, to compare their value directly is the only way because the two remaining paths cannot be excluded further by the necessary conditions. A numerical approach must be used to compare them, but we did not use such an approach. Instead, we derive some intuitively clear results related to the choice of optimal path in an analytical approach.

First, the following Lemma 3 is presented as a benchmark result:

Lemma 3

If $\rho = 1$, a big push from the initial state $n_0 = 0$ is desirable as long as there is efficient urbanization.

The proof is noted in i) of Appendix 3 Lemma 3 shows that a simple way to compare steady states in evaluating a big push is justified in the situation in which the government cares about the future of the country as much as the present. However, despite whatever long-run prospects the government has, the future is less evaluated than the present because the discount factor of time in this model displays the interest rate of ODA. Therefore, that the entire process of a big push should be evaluated under a positive time-discount value is important from a political perspective.

As for the optimal path selection and the value of the time discount, the following

Proposition 1 is derived as a result.

Proposition 1

Suppose that $n_0 = 0$ is given for an initial state and that an efficient steady state exists. At that time, there is a threshold level of discount factor represented by $\hat{\rho} \in (0, 1)$; a big-push is desirable when ρ is smaller than $\hat{\rho}$. In addition, $d\hat{\rho}/dg > 0$ holds.

The proof is noted in Appendix 3. According to Proposition 1, the government of a developing country should lead the economy to the urbanized steady state E only when the time discount is sufficiently small. Furthermore, the definition $\rho = 1/(1 + r)$ implies that donor countries should set a low interest rate for the receiving country to develop. Under an appropriately low interest rate, the cost for urbanization becomes lower than its benefit of that; consequently, governments of less-developed countries choose to promote urbanization. In addition, urbanization is likely to be desirable if rural sector productivity is low because the opportunity cost for urbanization is low during that time. However, this result depends on the assumption of small openness. An agricultural sector with low productivity requires a large labor input to feed the people if we consider a closed economy.

3.4 Optimal sequence of tax and subsidy

The final part of this paper presents discussion related to the desirability of intervention by government with taxes and subsidies by comparing the optimal path with the laissez-faire path. In the analysis of the present subsection, $n_0 = 0$ is given for the initial state. Furthermore, it is presumed that ρ is appropriately large so that the urbanization is desirable. Under such an assumption, we highlight the optimal political intervention to control an urbanization process.

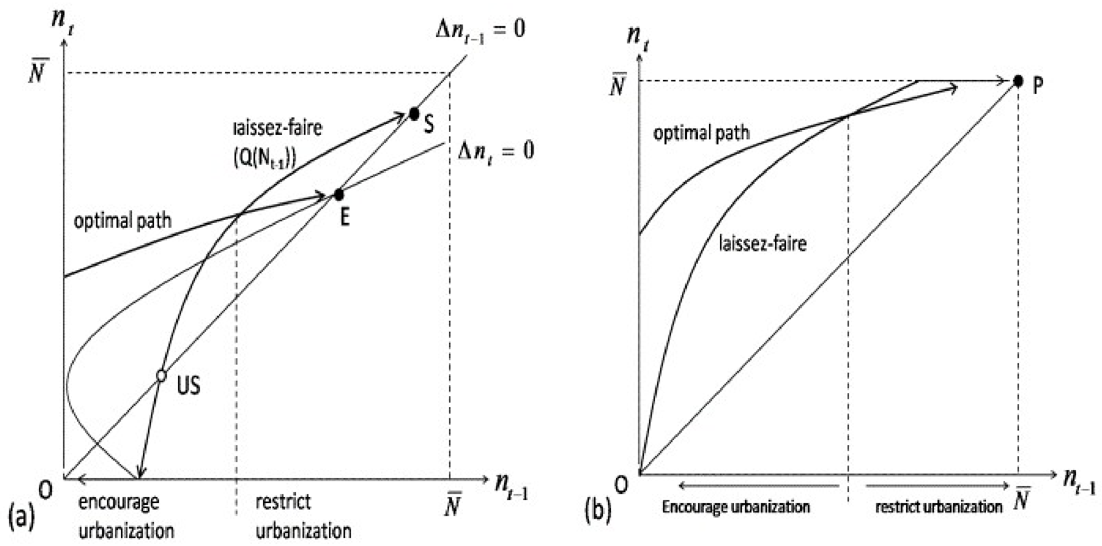

Figure 3 Optimal intervention

The laissez-faire and optimal urbanization paths are compared in Figs. 4(a) and 4(b). Under laissez-faire, the population of the urban sector is determined using a function $n_t = Q(n_{t-1})$, and the laissez-faire path of n_t is represented by the $n_t = Q(n_{t-1})$ curve, which is concave with a positive tangent, as depicted in Fig. 1(a). On the other hand, the optimal urbanization path is expressed as an upward sloping curve with a positive intersection [12].

We first regard Fig. 3(a). Both the laissez-faire and optimal paths intersect the 45 line in this figure. Because people concentrate in the urban sector until the per-capita productivity of that sector decreases to g and the urban sector is thereby overpopulated under laissez-faire, S is northeast of E. In Fig. 3(a), optimal n_t is smaller than that of laissez-faire when n_{t-1} is small, and is larger when n_{t-1} is close to N_E. That result implies the following: if $n_0 = 0$ is given as the initial state, the government should

[12] In Figs. 3(a) and 3(b), because we neglect the discreteness of this model and treat the model as if it were continuous, the tracks of n_t are drawn as smooth curves.

give subsidies to the urban sector in the early stage of development, and impose taxes when the economy is sufficiently developed [13]. In other words, urbanization should be initially encouraged, then restricted. The political implications near the steady state support the result of Henderson [3]: that cities become too large. However, this study supports that immigration in the urban sector should not always be restricted. Necessarily, policy changes must be made according to the stage of development if the process of development is considered.

Figure 3(b) portrays a polar case in which $g = 0$ holds and the internal steady state E is crowded out of the bounds of total population. In the figure, some objectives of urbanization control are emphasized. Furthermore, in that figure, the implications for optimal policy are presented as similar to those of the previous case; the urban sector should be subsidized initially, with taxes imposed during the latter stage of development. In the situation shown in Fig. 3(b), however, the economy automatically begins urbanization and converges to the optimal steady state P under laissez-faire [14]. Why should the government intervene in the development process?

Another purpose for intervention in the urbanization process is especially emphasized in Fig. 3(b): to control the speed of urbanization. In the analysis addressing the converging process of the model, the purpose of political intervention in this case is not only to lead the economy to the optimal equilibrium, unlike the arguments of big push. At the initial stage of development, urbanization should be accelerated so that the economy develops faster. However, during the latter half of development, rural-urban income disparities expand and immigration in the urban area is so greatly accelerated. There, urbanization should be slowed down for a soft landing on the long-run steady state.

[13]We can not strictly exclude the situation in which the optimal and laissez-faire paths might intersect more than three times, unlike Fig. 2(a) (the number of intersections must be odd). If so, the result becomes slightly more complicated, but the political implication of the model is almost the same.

[14]Because $n_t = 0$ is an unstable steady state, a small perturbation causes urbanization in that state.

Conclusively, optimal intervention to urbanization has the following four phases: i) big push for take-off, ii) acceleration of urbanization, iii) slowing of urbanization, and iv) completion of the optimal steady state. In the first and second phases of the sequence described above, the urban sector is subsidized, but it is taxed in the third and fourth phases.

4 Concluding remarks

This study has investigated the optimal urbanization process of an underdeveloped economy with a simplified dynamic model. In the present model, the combination of an intertemporal positive externality of production and simultaneous negative externality by congestion in the urban sector are important factors. That combination yields a trade-off between growth and congestion of the urban sector and the government controls rural-urban migration with consideration of the effects of a positive discount factor. In addition to steady states and long-run equilibria, the growth process was evaluated to determine the optimal dynamic policy in this study.

Even with extreme simplification of the model, this study has presented some noteworthy political implications by considering the process by which the urban sector grows. First, a big push is desirable only when the time discount is appropriately small, even though such a policy leads the economy to the better steady state. In other words, the ODA must be funded at an appropriately low interest rate to let the government of an underdeveloped economy resolve urbanization. Second, to optimize rural-urban migration, the urban sector should be subsidized to encourage immigration in that sector during early development, while it should be taxed to restrict immigration when the urban sector has grown sufficiently. In that context, the importance of controlling the speed of urbanization was also addressed.

The former and the latter results were derived by reconsidering well-known results of Murphy et al. [11] and Henderson [3], respectively, from a dynamic point of view. In fact, their implications are not justified in any situation by the results of our analysis. This study is not intended to contradict their contributions, but merely to remake and develop them. Future studies should develop the present model by explicit introduction of both private and public capital into the model, and by considering perpetual growth of the whole economy.

Appendix 1. Proof of Lemma 1

We presume that $F(n_{t-1})$ denotes the maximized current value of profit of urban sector when n_{t-1} is given in period t, and that the optimal n_t for given n_{t-1} is denoted as a function $\phi(n_{t-1})$. Because the functions $F(n_{t-1})$ is not affected directly by t, objective function of the government is represented by a function of n_{t-1} and n_t as $n_{t-1}^{\delta} n_t - bn_t^2 - gn_t + \rho F(n_t)$. The following first and second order conditions hold respectively if n_t is optimal and in $(0, 1)$;

$$n_{t-1}^{\delta} - 2bn_t - g + \rho F'(n_t) = 0 \tag{A.1}$$

$$-2b + \rho F''(n_t) < 0 \tag{A.2}$$

Differentiating $(A.1)$ by n_t and n_{t-1} yields

$$\delta n_{t-1}^{\delta-1} dn_{t-1} + [-2b + F''(n_t)]dn_t = 0. \tag{A.3}$$

And then, by substituting $(A.2)$ into $(A.3)$,

$$\frac{dn_t}{dn_{t-1}} = \frac{\delta n_{t-1}^{\delta-1}}{2b - F''(n_t)} > 0 \tag{A.4}$$

holds nearby the internal optimal path. Furthermore, $dn_t/dn_{t-1} = 0$ holds if optimal n_t is equal to 1 or 0. Therefore, if the optimal n_t for given n_{t-1} is denoted as a function $\phi(n_{t-1})$ which is not affected by time t, $\phi'(n_{t-1}) \geq 0$ holds.

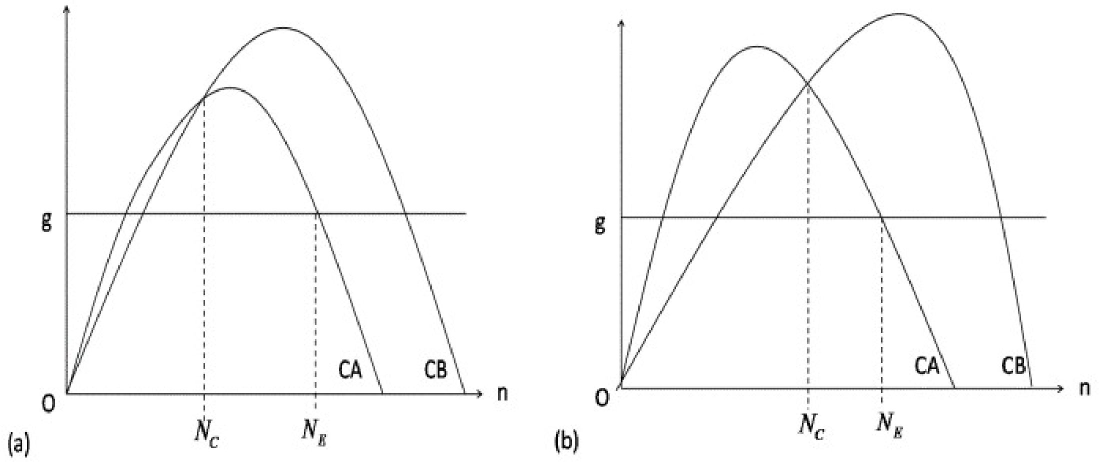

Figure A1 Efficiency of urbanized steady state

In light of the result presented above, the following are readily apparent. i) $n_{t+1} = \phi(n_t) \geq \phi(n_{t-1}) = n_t$ holds if $n_t > n_{t-1}$ holds, ii) $n_{t+1} = \phi(n_t) \leq \phi(n_{t-1}) = n_t$ holds if $n_t < n_{t-1}$, iii) $n_{t+1} = \phi(n_t) = \phi(n_{t-1}) = n_t$ holds if $n_t = n_{t-1}$. Q.E.D.

Appendix 2. A detailed derivation of the efficiency condition

At a steady state $n_{t-1} = n_t = n$, the per-capita income of urban sector in each period is

$$y(n, n) = n^{\delta} - bn \qquad (A.5)$$

In Figs. A1(a) and A1(b), the left-hand-side of eq. (10) and $y(n, n)$ are depicted respectively as curves CA and CB.

The two curves intersect twice at $n = 0$ and $n = N_C \equiv (\rho\delta/b)^{1/(1-\delta)} > 0$. The tangent of CB is positive at $n = N_C$, while that of CA at $n = N_C$ is positive if parameters hold for the following but are negative otherwise.

$$\rho \leq 1/(2-\delta) \tag{A.6}$$

i) If the tangent of CA is positive at $n = N_C$, as drawn in Fig. A1(a), then $y(N_E, N_E) \geq g$ holds as long as eq. (10) has real solutions.

ii) On the other hand, if the tangent of CA is negative at $n = N_C$, as drawn in Fig. A1(b), then $y(N_E, N_E) \geq g$ holds if and only if $y(N_C, N_C) \geq g$ holds. In fact, $y(N_C, N_C) \geq g$ is satisfied if the following parameters hold.

$$g > (1-\rho\delta)(\rho\delta/b)^{\delta/(1-\delta)} \tag{A.7}$$

Appendix 3. Proof of Lemma 3 and Proposition 1

The converging path to the steady state E and O are represented by $\mathrm{p_E}$ and $\mathrm{p_O}$ respectively. If an initial state n_0 is given, the aggregate present value GDP yield by path $\mathrm{p_E}$ and $\mathrm{p_O}$ are denoted respectively as $F_E(n_0, \rho)$ and $F_O(n_0, \rho)$. In addition, their difference $F_E(n_0, \rho) - F_O(n_0, \rho)$ is defined as a function $F_{EO}(n_0, \rho)$, which is the aggregate profit of urbanization.

If $n_0 = 0$ is given as an initial state and if the population distribution on the path $\mathrm{p_E}$ beginning at that initial state is represented by ψ_t, the following holds;

$$F_{EO}(0, \rho) = \Sigma_{t=1}^{\infty} \rho^{t-1} f_{EO}(\psi_{t-1}, \psi_t) = \Sigma_{t=1}^{\infty} \rho^{t-1} \left[\psi_{t-1}^{\delta}\psi_t - b\psi_t^2 - g\psi_t\right]. \tag{A.8}$$

In the above equation, $f_{EO}(\psi_{t-1}, \psi_t)$ represents the momentary profit of the urban sector in period t.

i) We consider a path $\mathrm{p_e}$ instead of $\mathrm{p_E}$, where $n_t = N_E$ holds for all $t \geq 1$, and denote the aggregate value of GDP by $F_e(n_0, \rho)$. It might be readily apparent that $F_e(n_0, \rho) \leq F_E(n_0, \rho)$ holds. If the steady state E is strictly efficient, or if $y(N_E, N_E) > g$ holds, then

$$F_e(0,1) - F_O(0,1) = -bN_E^2 + \Sigma_{t=2}^{\infty} \left[y(N_E, N_E)N_E - N_E g\right] > 0. \tag{A.9}$$

From inequality $(A.9)$ and Lemma 2, $F_{EO}(0,1) > 0$ holds if $g < (1-\delta)(\delta/b)^{\delta/(1-\delta)}$ holds.

ii) In addition, when $\rho = 0$, $F_O(0,0) = g$ and $F_E(0,0) = -b\psi_t^2 + (\bar{N} - \psi_t)g$; hence $F_{EO}(0) < 0$ holds. Thereby, Lemma 3 is proved.

iii) Because $F_E(n_0, \rho)$ and $F_O(n_0, \rho)$ are continuous in ρ, and because $F_{EO}(0,1) > 0$ and $F_{EO}(0,0) < 0$ hold, there is a $\hat{\rho} \in (0,1)$ which holds that $F_{EO}(n_0, \hat{\rho}) = 0$ in $0 \leq g < (1-\delta)(\delta/b)^{\delta/(1-\delta)})$.

iv) The differentiation of $F_{EO}(n_0, \rho)$ by ρ with the envelope theorem implies that

$$\frac{\partial F_{EO}(n_0, \rho)}{\partial \rho} = \Sigma_{t=2}^{\infty}(t-1)\rho^{t-2}\left(\psi_{t-1}^{\delta}\psi_t - b\psi_t^2 - \psi_t g\right). \tag{A.10}$$

In that time, we represent the path $\mathrm{p_E}$ by a function $\phi(n_{t-1})$ as the notation in Appendix 1, then $\phi(\psi_{t-1}) = \psi_t$ holds. If the economy is on the path $\mathrm{p_E}$, the current value of the aggregate profit in period t is represented by $F_{EO}(n_{t-1}, \rho)$ where n_{t-1} is given. The differentiation of $F_{EO}(n_{t-1}, \rho)$ by n_{t-1} with the envelope theorem yields

$$\frac{\partial F_{EO}(n_{t-1}, \rho)}{\partial n_{t-1}} = \frac{\partial f_{EO}(n_{t-1}, \phi(n_{t-1}))}{\partial n_{t-1}} = \delta n_{t-1}^{\delta-1}\phi(n_{t-1}) > 0. \tag{A.11}$$

Therefore, the momentary profit of urbanization in period t increases in n_{t-1}, which means that $f_{EO}(\psi_{t-1}, \psi_t)$ increases in time t if the economy is on the path $\mathrm{p_E}$ beginning at $n_0 = 0$. Furthermore, $F_{EO}(0, \hat{\rho}) = 0$ holds from the definition of $\hat{\rho}$. If we substitute those two facts and $(A.8)$ into $(A.10)$, $\partial F_{EO}(0,\rho)/\partial\rho|_{\rho=\hat{\rho}} > 0$ is confirmed because $-\Sigma_{t=1}^{T}\hat{\rho}^{t-1}f_{EO}(\phi_{t-1}, \phi_t) < 0$ holds for all finite T; hence $\hat{\rho}$ is unique if other parameters are given. In addition, $F_{EO}(0,\rho) < 0$ holds in $\rho > \hat{\rho}$ and $F_{EO}(0,\rho) > 0$ holds in $\rho < \hat{\rho}$ is proved for uniqueness of $\hat{\rho}$.

v) Finally, differentiating $(A.8)$with the envelope theorem yields the following;

$$dF_{EO}(0,\rho) = \left(-\Sigma_{t=1}^{\infty}\rho^{t-1}\psi_t\right)dg + \frac{\partial F_{EO}(0,\rho)}{\partial \rho}d\rho \tag{A.12}$$

If we set $dF_{EO}(0,\rho) = 0$, $d\hat{\rho}/dg|_{\rho=\hat{\rho}} > 0$ is proved because of $\partial f_{EO}(\rho)/\partial\rho|_{\rho=\hat{\rho}} > 0$. Q.E.D.

References

[1] L. Bertinelli, D. Black, Urbanization and Growth, Journal of Urban Economics 56 (2004) 80-96.

[2] J. C. H. Fei, G. Ranis, A theory of economic development, American Economic Review (1961) 533-565.

[3] J. V. Henderson, The sizes and types of cities, American Economic Review 64 (1974) 640-656.

[4] T. Iwaisako, Technology choice and patterns of growth in an overlapping generation model, Journal of Macroeconomics 24 (2002) 211-231.

[5] Y. Kanemoto, Theories of Urban Externalities, North-Holland, Amsterdam.

[6] Y. Kanemoto, T. Ohkawara, T. Suzuki, Agglomeration economies and a test for optimal city size in Japan, Journal of the Japanese and International Economies 10 (1996), 379-398.

[7] W. A. Lewis, Economic development with unlimited supplies of labour, Manchester School of Economic and Social Studies 22 (1954) 139-191.

[8] R. E. J. Lucas, On the mechanism of economic development, Journal of Monetary Economics 22 (1988) 3-42.

[9] K. Matsuyama, The market size, entrepreneurship, and the big push, Journal of the Japanese and International Economies 6 (1992) 347-364.

[10] Y. Murata, Rural-urban interdependence and industrialization, Journal of Development Economics 68 (2002) 1-34.

[11] K. Murphy, A. Shleifer, R. Vishny, Industrialization and the big push, Quarterly Journal of Economics 97 (1989) 1003-26.

[12] Y. Papageorgiou, D. Pines, Externalities, indivisibility, nonreplicability, and agglomeration, Journal of Urban Economics 48 (2000) 509-535.

[13] P. M. Romer, Increasing Returns and long-run growth, Journal of Political Economy 94 (1986) 1002-1037.

[14] World Urbanization Prospects, 2005 Revision, United Nations Publications, 2006.