Globalization, Interregional and International Inequalities

Dao-Zhi Zeng∗ Laixun Zhao†

November 2007

Abstract

This paper examines interregional and international inequalities in a setup of two countries and four regions. Different from the existing literature, countries and regions are not required to be symmetric in size. Capital but not labor is mobile across regions and countries. We find that the interregional and international inequalities are closely related to globalization and the efficiency of local governance. In other words, they are jointly determined by the domestic transport costs (e.g., infrastructure, administrative barriers, etc) in the two countries and the international trade cost. Particularly, the interregional inequality may be either a monotonically increasing or an inverted U-curve function of its own domestic transport costs. Also, the interregional inequality decreases with the national manufacturing share. These results shed light on the so-called “deindustrialization” phenomenon.

Key Words: Regional Inequality, Firm Location, Infrastructure, Governance, Deindustrialization

JEL Classification: R12

∗Graduate School of Management, Kagawa University, Saiwai-Cho 2-1, Kagawa 760-8523, Japan. E-mail: zeng@gsm.kagawa-u.ac.jp.
†Research Institute for Economics and Business, Kobe University, Kobe 657-8501, Japan. E-mail: zhao@rieb.kobe-u.ac.jp.
1 Introduction

There is deep concern that the recent wave of globalization has enlarged the inequality between developed and developing countries, and between rich and poor regions within a country. Some developed countries as well as regions are concerned that globalization might affect negatively their ability to sustain high living standards, while developing countries and regions fear that a rise in the living standards of rich countries and regions is at the expense of the poor.

Interregional and international inequalities are evidently related to interregional and international trade. For example, as a result of economic liberalization policies in the past 30 years, China has narrowed its distance with advanced countries. However, the unequal development between the coastal regions and the inland is a major concern of the government. According to Fujita and Hu (2001), during the period between 1985 and 1994, the absolute gap between coastal and interior China increased by 10 times (p.8). They predicted that “as long as globalization and economic liberation continue in China, production agglomeration toward the coastal area will continue” (p.31). In many cases, such inequalities are complicated, and they usually do not change in a monotonic way with economic policies. For example, Petrakos and Saratsis (2000) find that the regional inequalities in Greece have a cyclical character: increasing in periods of economic expansion and decreasing in periods of economic recession. In addition, Sánchez-Reaza and Rodríguez-Pose (2002) find that in the case of Mexico, the proximity to the US market has been a determinant in the concentration of economic activity, and these forces have interacted with uneven distribution of infrastructure and public services to create very different opportunities for different regions. They further observe that the end of the ISI (import substitution industrialization) period was characterized by regional convergence, whereas economic liberalization and integration have been connected by regional divergence. As for the case of developed countries, Davis and Weinstein (2005) find that the uneven pattern of manufacturing activity in Japan contributes to overall efficiency—if aggregate activity were to be spread evenly across all regions, output would be lowered by 5 percent.

The above phenomena have generated intense academic and policy debates as to their causes and policy remedies. Existing studies basically take two approaches to the problem. One is based on income/wage inequality, such as Feenstra and Hanson (1996), Freeman (1995) and Wood (1995), who highlight the role of import competition from develop-
ing countries, and Acemoglu (2002), Harrigan (1998), Krugman and Lawrence (1994), Lawrence and Slaughter (1993) who focus on the role of technological changes. The other approach is based on industrial activities and adopts spatial-economy models (e.g., Krugman and Venables, 1995; Fujita et al, 1999; Puga, 1999). This literature attributes the increasing inequality to increasing-returns-to-scale technology, monopolistic competition and transport costs in product markets. The two approaches are closely related because the latter one can be interpreted as the real wage inequality.

The present paper follows the second approach, which we believe is suitable for analyzing the so-called “deindustrialization” phenomenon, because it focuses on the concentration and relocation of industrial activities. The recent wave of globalization is characterized by freer mobility of goods as well as factors across countries, in the form of lower transportation costs, trade costs, easier access to foreign capital, etc. These are in turn driven by technological advances in transport and communications, and also partly by deliberate policy designs such as those in China and Russia. Such factors must be explicitly incorporated in order to address issues related to regional and international inequalities, yet by and large they have been neglected in the literature. While in the real world, one often hears loud cries of “deindustrialization”.¹ Our analysis will highlight the importance of capital mobility, which impacts both interregional and international inequalities in complicated ways.

For this purpose, we examine a spatial economy of two asymmetric countries and four asymmetric regions, with both domestic and international transport/trade costs. We have in mind two countries with close economic ties such as Canada and the U.S., Germany and Poland, Japan and South Korea, Mexico and the U.S., etc. Each country has its densely populated manufacturing centers and remote countryside, with interregional administrative transport costs and international trade barriers. We wish to use this setup to analyze how globalization in the form of higher capital mobility across countries as well as improvements of domestic infrastructure and administrative barriers affect the interregional and international income inequality. In particular, we ask the following questions: Does globalization lead to agglomeration in the larger region in the larger country? Will the remote countryside of the smaller country be deserted? Is the so-called ‘deindustrialization’ of some developed countries related to the domestic reduction

¹Also called ‘the giant sucking sound’ by former U.S. presidential candidate Ross Perot, referring to U.S. firms moving out to Mexico, and ‘hollowing out’ in Japan, referring to Japanese firms’ outsourcing activities to Southeastern Asian countries.
of administrative barriers and transport costs, or the international trade costs? If yes, in what way? Can one country’s efficiency of governance affect the regional inequalities of the other country?

With our setup, we obtain the following main results. I. If capital is not mobile across countries, then i) the interregional inequality in a country is determined by its own domestic transport costs which in turn depends on its administrative and infrastructure efficiency, and by the international trade costs. It is independent of the transport costs in the other country; ii) regional inequality rises monotonically with globalization. And a larger national manufacturing share brings a lower interregional inequality. This result explains why many developing countries actively adopt various and systematic policies to attract foreign direct investment, especially in manufacturing.

II. In contrast, if capital is mobile across countries, i) the regional inequalities and the international inequality are jointly determined by the transport costs in both countries and the international trade costs. A change in one country’s domestic transport cost (e.g., infrastructure, governance efficiency) affects the incentives of production, causing capital to move not only domestically but also internationally, which further leads to changes of industrial relocation in both countries. If the domestic transport costs are equal in the two countries, then the regional inequality in the bigger country evolves as an inverted U-curve, while that in the smaller country takes a monotonically increasing form with respect to the trade costs; ii) A country’s national firm share rises (falls) with a lower domestic transport cost of its own (the other country). Consequently, facing the wave of globalization (in contrast to the case of no international capital mobility), countries with poor infrastructure or low governance efficiency may lose.

Earlier models consider two regions or countries, and analyze either (domestic) regional inequalities or international inequalities but not both simultaneously, see for instance Krugman and Venables (1995), Puga (1999) and Fujita et al. (1999). Meanwhile, Krugman and Livas Elizondo (1996) and Paluzie (2001) employ three-location models with two regions in one country and the rest of the world as the third region. Using land rent and commuting costs as centrifugal forces, the first study shows that closed markets promote interregional agglomeration while open markets discourage it. The authors predict that economic liberalization in developing countries will shrink the interregional inequalities. To the contrary, based on immobile consumers as centrifugal forces, Monfort and Nicolini (2000) and Paluzie (2001) predict that economic integration exacerbates the
interregional inequalities monotonically. The stark difference in their conclusions stimulated our interest in the present paper to examine more closely the inequality issue.

Mainly for the ease of tractability, all the existing two-country-four-region models assume that the two countries are of the same size, and the regions are also symmetric. In Monfort and Nicolini (2000), the domestic transport costs in the two countries are also kept the same. Consequently, an infrastructure improvement in one country automatically implies the same improvement in the other country. Symmetry also renders these models unable to explore the relation between the national manufacturing share and the interregional inequality. Thus they find that the domestic geographies of the two countries are independent.

Our setup differs from the existing models in a number of important ways. First, neither the regions nor the countries are symmetric. Normally, the asymmetry would create tremendous difficulty in solving the model. But we are able to obtain analytical solutions in the present paper. More importantly, it enables us to analyze unequal development between regions and across countries, which are absent in the existing literature. Second, we allow capital to move across countries, an important characteristics of globalization. That is, capital seeks the highest-profit opportunities, wherever they are. This assumption enables us to exam issues related to the so-called ‘deindustrialization’ of manufacturing industries, as often heard in the media. Third, while Behrens et al. (2006a, 2006b, 2007) use a quasi-linear utility function, we adopt a standard Cobb-Douglas utility function, as is often used in models of monopolistic competition. Monfort and Nicolini (2000) and Paluzie (2001) also use Cobb-Douglas utility functions, but important results are obtained with simulations.

The rest of paper is structured as follows. Section 2 presents the basic model; Section 3 derives and discusses the location equilibria. The first part considers the case of no international capital mobility and the second part incorporates international mobility. Section 4 concludes.

2 The Basic Model

We consider a spatial economy of two countries. In each country, there exist two regions: large and small. The two regions in the same country have the same physical geographical constraints except their population size, while regions in different countries may also differ
in transport costs. There are two sectors in each country: manufacturing and a numéraire sector (e.g., agriculture). Two factors, labor and capital, are used in production. Capital but not labor is mobile across countries.

As an important feature of this paper, the two countries differ in size, i.e., their endowments of capital and labor are both different. We assume Country 1 to be bigger than Country 2, and region 1 to be bigger than region 2 in both countries. More precisely,

\[
\frac{L_1}{L} = \Theta \left( > \frac{1}{2} \right), \quad \frac{l_{11}}{L_1} = \theta_1, \quad \frac{l_{21}}{L_2} = \theta_2, \quad \left( \theta_1, \theta_2 > \frac{1}{2} \right),
\]

where the first subscript denotes country, and the second one denotes region. Thus, \( L \) is the world total endowment of labor, \( L_i \) is that of country \( i \), and \( l_{ij} \) is the labor endowment of country \( i \) in region \( j \). For short, we also call region \( j \) in country \( i \) by region \( ij \). Finally, \( \Theta, \theta_1 \) and \( \theta_2 \) are the respective shares of country 1, region 1 in country 1 and region 1 in country 2. We assume them to lie within suitable values to ensure that every region is involved in agricultural production.

The above definition implies

\[
l_{11} = \Theta \theta_1 L, \quad l_{12} = \Theta (1 - \theta_1) L, \quad l_{21} = (1 - \Theta) \theta_2 L, \quad l_{22} = (1 - \Theta) (1 - \theta_2) L.
\]

For convenience, we subsequently use \( i' \) to indicate the country other than \( i \), and \( j' \) to indicate the region other than \( j \).

### 2.1 Preferences

Individuals in the two countries are identical and share the same Cobb-Douglas tastes for the two types of goods expressed by utility function

\[
U = \frac{1}{\mu^\mu (1 - \mu)^{1-\mu} C_M^{\mu} C_A^{1-\mu}},
\]

where \( C_M \) represents a composite index of the consumption of the \( M \)-sector goods, \( C_A \) is the consumption of the \( A \)-sector good, and \( \mu \in (0, 1) \) is a constant denoting the expenditure share of manufactured goods. The quantity index \( C_M \) is defined by a constant-elasticity-of-substitution (CES) function over a continuum of varieties of manufactured
goods
\[ C_M = \left( \int_0^N c_i^{1-\frac{1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}, \]
where \( N \) is the number of varieties, \( c_i \) is the consumption of each available variety \( i \). Parameter \( \sigma > 1 \) is the constant elasticity of substitution between any two varieties and \( 1 - 1/\sigma \) is the intensity of the preference for variety in manufactured goods.

2.2 Transportation

As in the standard literature of spatial economics, the agricultural product is freely transported between the two countries. We use it as the numéraire good and assume that no country alone has enough labor to satisfy the world demand for good \( A \). Then the prices of the agricultural good are equalized internationally: \( p_A = p_A^* = 1 \).

In the manufacturing sector, transaction costs across both regions and countries are formalized as Samuelson iceberg costs. Specifically, \( t_i > 1 \) units of the manufactured good must be shipped for one unit to reach the other region in country \( i \). In other words, a fraction \( t_i - 1 \) is lost in domestic transportation of country \( i \). Similarly, \( \tau \) units must be transported across countries for 1 unit to arrive, i.e., \( \tau - 1 \) is lost in international transportation. International transaction costs are assumed to be independent of the regions involved. The costs are shown in Figure 1.

![Figure 1: Two countries and four regions](image)

Following Behrens et al. (2006a, 2006b, 2007), we call \( t_1 \) and \( t_2 \) the (domestic) transport costs, and \( \tau \) the (international) trade cost. Furthermore, we assume that interna-
tional transaction is more costly than domestic transaction because trade costs might include a fraction of tariffs: \( \tau \geq t_i \ (i = 1, 2) \).

Because of transaction costs, the price in region \( ij \) of a representative manufacturing good produced in region \( i_1j_1 \) is, where \( i \) identifies the country and \( j \) identifies the region,

\[
p_{i_1j_1 \rightarrow ij} = \begin{cases} 
  p_{ij} & \text{if } i_1 = i, \ j_1 = j \\
  p_{i_1j_1} t_i & \text{if } i_1 = i, \ j_1 \neq j \\
  p_{i_1j_1} \tau & \text{if } i_1 \neq i.
\end{cases}
\]

Then the CES demand in region \( ij \) for the manufacturing good \( s \) produced in region \( i_1j_1 \) is

\[
d_{i_1j_1 \rightarrow ij}(s) = \frac{p_{i_1j_1 \rightarrow ij}^{-\sigma}}{p_{ij}^{1-\sigma}} \mu I_{ij},
\]

where \( I_{ij} \) is the total income in region \( ij \),

\[
P_{ij} = \sum_{i_1=1}^{2} \sum_{j_1=1}^{2} \left( \int_{s \in n_{i_1j_1}} p_{i_1j_1 \rightarrow ij}(s)^{1-\sigma} ds \right)^{\frac{1}{1-\sigma}},
\]

is the price index in region \( ij \), and \( n_{i_1j_1} \) is the number of varieties in region \( i_1j_1 \). The total demand for a manufacturing good \( s \) produced in region \( ij \) is the following sum:

\[
d_{ij} = \sum_{i_1=1}^{2} \sum_{j_1=1}^{2} d_{ij \rightarrow i_1j_1}.
\]

2.3 Production

The representative consumer owns one unit of capital which can be invested, in either country. The world total endowment of capital is thus \( L = L_1 + L_2 \).

We have assumed no Ricardian comparative advantage in production across regions. In other words, productivities are identical across countries and regions in either the agricultural sector or the manufacturing sector. In particular, we assume one worker can produce one unit of the agricultural good. Then the wages in the two countries are equalized at: \( w_1 = w_2 = 1 \).

Firms are monopolistically competitive in the manufacturing sector, under which each firm produces a single differentiated good. The production of manufactures features economies of scale. Each firm in region \( ij \) requires one unit of capital as a fixed input.
and \((\sigma - 1)/\sigma\) workers as a marginal input. The total number of firms is then \(N = L\). To produce \(q\) units of output, the total input of a firm in region \(ij\) is

\[
    r_{ij} + \frac{\sigma - 1}{\sigma} q,
\]

where \(r_{ij}\) is the unit return of capital (i.e., profits) in region \(ij\).

A typical firm in region \(ij\) determines prices by maximizing profits

\[
    \Pi_{ij} = p_{ij}d_{ij\rightarrow ij} + p_{ij\rightarrow ij'}d_{ij\rightarrow ij'} + p_{ij\rightarrow i'1}d_{ij\rightarrow i'1} + p_{ij\rightarrow i'2}d_{ij\rightarrow i'2}
    - \frac{\sigma - 1}{\sigma} w_i[d_{ij\rightarrow ij} + td_{ij\rightarrow ij'} + \tau(d_{ij\rightarrow i'1} + d_{ij\rightarrow i'2})] - r_{ij},
\]

where \(i' \neq i, j' \neq j\). According to (1), the first-order condition gives

\[
    p_{ij\rightarrow ij} = \begin{cases} 
1 & \text{if } i_1 = i, j_1 = j, \\
t_i & \text{if } i_1 = i, j_1 \neq j, \\
\tau & \text{if } i_1 \neq i.
\end{cases}
\]

(3)

Therefore, the price index is simplified as

\[
    P_{ij} = [k_{ij} + \phi_t \mu k_{ij} + \phi_\tau (k_{i'1} + k_{i'2})]^\frac{\phi_t}{\phi_\tau},
\]

where \(k_{ij}\) is the number of firms in region \(ij\), and \(\phi_t = t_i^{1-\sigma} \in (0, 1]\) and \(\phi_\tau = \tau^{1-\sigma} \in (0, 1]\) are the domestic and international trade ‘freeness’. That is, a larger \(\phi\) means smaller costs.

Free entry in production ensures zero profits. Thus, (2) and (3) give

\[
    r_{ij} = \frac{1}{\sigma}[d_{ij\rightarrow ij} + td_{ij\rightarrow ij'} + \tau(d_{ij\rightarrow i'1} + d_{ij\rightarrow i'2})].
\]

(4)

Meanwhile, given price (3), the CES demands of representative varieties are then determined as

\[
    d_{ij\rightarrow ij} = \frac{\mu I_{ij}}{k_{ij} + k_{ij'} \phi_t + (k_{i'1} + k_{i'2}) \phi_\tau},
\]

\[
    d_{ij'\rightarrow ij} = \frac{t_i^{-\sigma} \mu I_{ij}}{k_{ij} + k_{ij'} \phi_t + (k_{i'1} + k_{i'2}) \phi_\tau},
\]

\[
    d_{i'j\rightarrow ij} = \frac{\tau^{-\sigma} \mu I_{ij}}{k_{ij} + k_{ij'} \phi_t + (k_{i'1} + k_{i'2}) \phi_\tau},
\]

\[
    d_{i'j'\rightarrow ij} = \frac{t_i^{-\sigma} \mu I_{ij}}{k_{ij} + k_{ij'} \phi_t + (k_{i'1} + k_{i'2}) \phi_\tau},
\]
\[
d_{i'j'\rightarrow ij} = \frac{\tau^{-\sigma} \mu I_{ij}}{k_{ij} + k_{ij}' \phi t_i + (k_{i'1} + k_{i'2}) \phi r}.
\]

### 2.4 Capital mobility

Physical capital moves in search of the highest nominal reward.\(^2\) Denote the capital (firm) share of Country 1 by \(\Lambda\), and that of region 1 inside Country 1 by \(\lambda_1\) and the counterpart of region 1 inside country 2 by \(\lambda_2\). Then

\[
k_{11} = \lambda_1 \Lambda N, \quad k_{12} = (1 - \lambda_1) \Lambda N, \quad k_{21} = \lambda_2 (1 - \Lambda) N, \quad k_{22} = (1 - \lambda_2)(1 - \Lambda) N.
\]

Following established tradition in economic geography, we assume that markets for goods adjust instantaneously, while international migration of capital is relatively slow, implying that wages adjust much faster than the capital share. Also following the literature, we apply a standard dynamic system to describe the international factor flow:

\[
\dot{\lambda}_i = (r_{1i} - r_{2i}) \lambda_i (1 - \lambda_i), \quad (5)
\]

\[
\dot{\Lambda} = (r_1 - r_2) \Lambda (1 - \Lambda). \quad (6)
\]

There are two phases. (5) describes the interregional mobility and (6) describes the international mobility. Both are adopted from replicator dynamics, routinely used in evolutionary game theory (Weibull, 1995, p. 73), and is also used in standard textbooks such as Fujita et al. (1999, p. 62) and Baldwin et al. (2003, p. 72). We assume that the international adjustment in the second phase starts after the domestic adjustment in the first phase has finished so that \(r_{11} = r_{12} \equiv r_1\) and \(r_{21} = r_{22} \equiv r_2\).

---

\(^2\)To simplify the calculation of the regional income, we make the following assumption: the capital in each region originates uniformly from the four regions. That is, regardless of the spatial allocation of industry and the degree of openness, \(\Theta \theta_1\) (resp. \(\Theta(1 - \theta_1),\) \((1 - \Theta)\theta_2,\) and \((1 - \Theta)(1 - \theta_2)\)) of the capital in each region originates from region 11 (resp. 12, 21, and 22). This is the four-region version of the standard assumption imposed for two-region models (see Baldwin et al. (2003, p. 74). This assumption does not change the equilibrium shares of capital but simplifies the mathematics a lot.
3 Equilibrium

3.1 Immobile capital across countries

We start with a benchmark case of no capital mobility across countries. This first phase determines domestic shares $\lambda_1$ and $\lambda_2$, given $r_{11} = r_{12} (\equiv r_1)$ and $r_{21} = r_{22} (\equiv r_2)$.

Without international capital mobility, Country 1’s national share of capital/firms $\Lambda$ is fixed. If $\Lambda = 1$ or $0$, then the model degenerates to the case of one country with two regions, which is widely seen in the literature. Therefore, we assume that $\Lambda \in (0, 1)$.

Equilibrium in Country 1 requires that $r_{11} = r_{12}$, which can be rewritten as

$$d_{11 \rightarrow 11} + t_1 d_{11 \rightarrow 12} + \tau(d_{11 \rightarrow 21} + d_{11 \rightarrow 22}) = d_{12 \rightarrow 12} + t_1 d_{12 \rightarrow 11} + \tau(d_{12 \rightarrow 21} + d_{12 \rightarrow 22}).$$

Because $d_{11 \rightarrow 21} + d_{11 \rightarrow 22} = d_{12 \rightarrow 21} + d_{12 \rightarrow 22}$, the above equation is simplified as

$$\frac{I_{11}}{(k_{11} + \phi_t k_{12}) + \phi_r (k_{21} + k_{22})} = \frac{I_{12}}{(k_{12} + \phi_t k_{11}) + \phi_r (k_{21} + k_{22})}.$$  \(7\)

Denote the average return of capital by

$$\bar{r} = \frac{1}{N} \sum_{i=1}^{2} \sum_{j=1}^{2} r_{ij} k_{ij}.$$

According to the uniform assumption of footnote 2, we have $I_{11} = \theta_1 \Theta L (1 + \bar{r})$ and $I_{12} = (1 - \theta_1) \Theta L (1 + \bar{r})$. Therefore, (7) yields

$$\lambda_1 = \theta_1 + \frac{2\theta_1 - 1}{1 - \phi_t} \left( \frac{1 - \Lambda}{\Lambda} \phi_r + \phi_t \right).$$  \(8\)

Since (8) is the only equilibrium, it is stable with respect to (5) according to Tabuchi and Zeng (2004).

Obviously if $\Lambda$ is close to 1, we have the case of a one-country-two-region model. In particular, (8) degenerates to the firm share (16) of Ottaviano and Thisse (2004). The property $\lambda_1 > \theta_1$ says that a larger region attracts a more-than-proportionate share of firms in a monopolistically competitive industry with increasing returns technology. This is the so-called regional home market effect (HME) (see e.g., Krugman, 1980; Helpman and Krugman, 1985).
Similarly, mobility of capital in country 2 requires that \( r_{21} = r_{22} \), which implies
\[
\frac{I_{21}}{k_{21} + k_{22}\phi_{t_2} + (k_{11} + k_{12})\phi_{r}} = \frac{I_{22}}{k_{22} + k_{21}\phi_{t_2} + (k_{11} + k_{12})\phi_{r}},
\]
(9)
and derives
\[
\lambda_2 = \theta_2 + \frac{2\theta_2 - 1}{1 - \phi_{t_2}} \left( \frac{\Lambda}{1 - \Lambda} \phi_{r} + \phi_{t_2} \right).
\]
(10)
This equilibrium is again stable due to its uniqueness. Since \( \Lambda \in (0, 1) \), firms in both countries produce positive outputs. And both \( [(1 - \Lambda)/\Lambda]\phi_{r} \) in (8) and \( [\Lambda/(1 - \Lambda)]\phi_{r} \) in (10) are positive; that is, the HME is strengthened due to international trade.

**Proposition 1** *When capital is immobile across countries,*

(i) a decrease in the domestic transport costs \( t_i \) increases the regional inequality of country \( i \);

(ii) a decrease in the international trade costs \( \tau \) increases the regional inequality of both countries.

(iii) a larger firm-share in a country decreases the regional inequality of this country.

**Proof:** We only consider the case of Country 1. The conclusion stems from the following facts
\[
\frac{\partial \lambda_1}{\partial \phi_{t_1}} = \frac{(2\theta_1 - 1)[\Lambda + (1 - \Lambda)\phi_{r}]}{\Lambda(1 - \phi_{t_1})^2} > 0,
\]
\[
\frac{\partial \lambda_1}{\partial \phi_{r}} = \frac{(2\theta_1 - 1)(1 - \Lambda)}{\Lambda(1 - \phi_{t_1})} > 0,
\]
\[
\frac{\partial \lambda_1}{\partial \Lambda} = \frac{\phi_{r}(2\theta_1 - 1)}{\Lambda^2(1 - \phi_{t_1})} < 0.
\]

\[
\square
\]

Part (i) of Proposition 1 is simply a restatement of the results obtained in the literature from a one-country and two-region model. Our model of 2 asymmetric countries and 4 asymmetric regions further confirms that the regional HME is strengthened when trade becomes freer.

Part (ii) shows the relationship between regional inequality and international trade costs: the openness of a country to the world is accompanied by regional inequality. It suggests that globalization is likely to lead to agglomeration in the larger region of the
larger country, and in the extreme, the remote countryside of the smaller country might be deserted. This result is consistent with some empirical studies, such as Sánchez-Reaza and Rodríguez-Pose (2002) and Rodríguez-Pose and Sánchez-Reaza (2005), who find unmistakable trends towards greater regional inequality and polarization, when Mexican economic policy shifted from a closed-economy approach to trade liberalization since the mid-1980s, and to economic integration since 1994. They explain that in Mexico, proximity to the US market has been a determinant in the concentration of economic activity, and these forces have interacted with uneven distribution of infrastructure and public services to create very different opportunities for different regions across Mexico.

Most importantly, part (iii) says that a larger national manufacturing share results in a lower interregional inequality. Intuitively, when the national manufacturing share is not too large, the competition effect in the bigger region is weak, then firms can benefit from relocating in the bigger region. This result explains why many developing countries actively adopt various and systematic policies to attract foreign direct investment, especially in manufacturing. Note that this result cannot be obtained if the countries are exogenously assumed symmetric, as done in the literature.

Mathematically, results (i) and (ii) depend on the international immobility of capital, from which Λ in (8) and (10) becomes a constant. In subsequent analysis we shall see that the manufacturing share is jointly determined by various transport costs when capital is internationally mobile. Therefore, according to (iii), the inequalities become much more complicated.

3.2 Internationally mobile capital and international inequality

After the domestic adjustments in the first phase, we are now ready to endogeneize Country 1’s capital share Λ. To do so, we relax the assumption of no capital mobility across countries and let capital move wherever nominal reward is the highest.

Previous equations (4), (7) and (9) give

\[
\begin{align*}
    r_1 &= \frac{\mu}{\sigma} \left[ \frac{(1 + \phi_{t_1})\theta_1\Theta L(1 + \bar{r})}{(k_{11} + k_{12}\phi_{t_1}) + (k_{21} + k_{22})\phi_r} + \frac{2\phi_r\theta_2(1 - \Theta)L(1 + \bar{r})}{(k_{21} + k_{22}\phi_{t_2}) + (k_{11} + k_{12})\phi_r} \right], \\
    r_2 &= \frac{\mu}{\sigma} \left[ \frac{(1 + \phi_{t_2})\theta_2(1 - \Theta)L(1 + \bar{r})}{(k_{21} + k_{22}\phi_{t_2}) + (k_{11} + k_{12})\phi_r} + \frac{2\phi_r\theta_1\Theta L(1 + \bar{r})}{(k_{11} + k_{12}\phi_{t_1}) + (k_{21} + k_{22})\phi_r} \right].
\end{align*}
\]
At equilibrium, \( r_1 = r_2 \equiv \bar{r} \) must hold. Then

\[
\frac{\sigma}{\mu} \bar{r}(1 + \phi_{t_1} - 2\phi_r) = \frac{(1 - \Theta)(1 + \bar{r})L\theta_2}{(k_{11} + k_{12}\phi_{t_1}) + (k_{21} + k_{22}\phi_r)(1 + \phi_{t_2}) - 4\phi_r^2},
\]

\[
\frac{\sigma}{\mu} \bar{r}(1 + \phi_{t_2} - 2\phi_r) = \frac{\Theta(1 + \bar{r})L\theta_1}{(k_{21} + k_{22}\phi_{t_2}) + (k_{11} + k_{12}\phi_r)(1 + \phi_{t_1}) - 4\phi_r^2}.
\]

Substituting (8) and (10) into the above yields:

\[
\frac{\sigma}{\mu} \bar{r}(1 + \phi_{t_1} - 2\phi_r) = \frac{(1 - \Theta)(1 + \bar{r})}{2\Lambda \phi_r + (1 - \Lambda)(1 + \phi_{t_2})}[(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2],
\]

\[
\frac{\sigma}{\mu} \bar{r}(1 + \phi_{t_2} - 2\phi_r) = \frac{\Theta(1 + \bar{r})}{2(1 - \Lambda)\phi_r + \Lambda(1 + \phi_{t_1})}[(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2],
\]

which imply the equilibrium share as

\[
\Lambda^* = \frac{\Theta[(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2] - 2\phi_r(1 + \phi_{t_2} - 2\phi_r)}{(1 + \phi_{t_1})(1 + \phi_{t_2} - 2\phi_r)} \quad (11)
\]

Due to uniqueness, this equilibrium is stable with respect to (5) according to Tabuchi and Zeng (2004). At the equilibrium \( \Lambda^* \), the capital return is \( r^* = \frac{\mu}{(\sigma - \mu)} \).

Expression (11) is independent of both \( \theta_1 \) and \( \theta_2 \). This is due to the assumption that the (international) trade costs are identical for transactions between any two regions across country borders.

If \( \phi_{t_1} = \phi_{t_2} = 1 \) (i.e., zero domestic trade cost), then the case degenerates to the traditional model of two countries (regions) again. Also, expression (11) holds as an interior equilibrium \( \Lambda \in (0, 1) \) only if

\[
\frac{2\phi_r(1 + \phi_{t_2} - 2\phi_r)}{(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2} < \Theta < \frac{(1 + \phi_{t_1})(1 + \phi_{t_2} - 2\phi_r)}{(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2}. \quad (12)
\]

For \( \Theta \) lying outside of this range, we have corner solutions. Specifically, for a larger \( \Theta \), Country 1’s capital share \( \Lambda \) becomes 1 (all firms agglomerate in Country 1), and for a smaller \( \Theta \), \( \Lambda \) becomes 0 (all firms agglomerate in Country 2).
Now rewrite (11) as
\[ \Lambda^* = \Theta + \Gamma + \Omega(\Theta - 1/2). \] (13)
where
\[ \Gamma \equiv \frac{(\phi_{t_1} - \phi_{t_2})\phi_{\tau}}{(1 + \phi_{t_1} - 2\phi_{\tau})(1 + \phi_{t_2} - 2\phi_{\tau})}, \]
\[ \Omega \equiv 2\phi_{\tau}\left(\frac{1}{1 + \phi_{t_1} - 2\phi_{\tau}} + \frac{1}{1 + \phi_{t_2} - 2\phi_{\tau}}\right). \]

Equation (13) shows that the national manufacturing share is determined by two effects. The expression \( \Gamma \) stems from domestic transport cost differences in the two countries, which shows the importance of relative infrastructure construction and administrative efficiency. We thus call it the infrastructure effect. Firms are likely to choose a country with lower transport costs and higher administrative efficiency. If Country \( i \) has better infrastructure, then \( \phi_{t_i} > \phi_{t_j} \) and more firms move in. This effect disappears if the domestic transport costs are equalized between the two countries, as in the existing literature. The third term in (13) is the international HME: the larger country is likely to hold a more-than-proportionate share of firms because of the increasing returns technology in manufacturing.

We can further calculate the partial derivatives from (11) as follows:
\[
\begin{align*}
\frac{\partial \Lambda^*}{\partial \phi_{t_1}} &= \frac{2(1 - \Theta)\phi_{\tau}}{(1 + \phi_{t_1} - 2\phi_{\tau})^2} > 0, \\
\frac{\partial \Lambda^*}{\partial \phi_{t_2}} &= -\frac{2\Theta\phi_{\tau}}{(1 + \phi_{t_2} - 2\phi_{\tau})^2} < 0, \\
\frac{\partial \Lambda^*}{\partial \phi_{\tau}} &= \frac{(2 + \phi_{t_1} + \phi_{t_2})(2\Theta - 1) + (\phi_{t_1} - \phi_{t_2})}{(1 + \phi_{t_1} - 2\phi_{\tau})(1 + \phi_{t_2} - 2\phi_{\tau})} \\
&\quad + \frac{4\phi_{\tau}(\phi_{t_1} - \phi_{t_2})[1 + \Theta\phi_{t_1} + (1 - \Theta)\phi_{t_2} - 2\phi_{\tau}]}{(1 + \phi_{t_1} - 2\phi_{\tau})^2(1 + \phi_{t_2} - 2\phi_{\tau})^2}. \tag{14}
\end{align*}
\]

They are summarized as:

**Proposition 2** (i) Country 1’s equilibrium firm-share \( \Lambda^* \) rises with a lower domestic transport cost \( t_1 \), but falls with a lower domestic transport cost \( t_2 \) in the smaller Country 2;

(ii) The relationship between the national firm share \( \Lambda^* \) and the international trade cost \( \tau \) depends on the domestic transport costs \( t_1 \) and \( t_2 \). When \( t_1 = t_2 \), lowering \( \tau \) increases...
Part (i) says that lowering the domestic transport costs in the two countries have completely different impacts on the international inequality. This arises because lowering domestic transport costs attracts more firms into the country from the other country. This result cannot be obtained when \( t_1 = t_2 \) is imposed exogenously, as in Monfort and Nicolini (2000). Part (ii) and expression (14) further show the importance of the infrastructure effect. If the larger country is a developing country and the smaller one is a developed country (like China and Japan), then \( \phi_{t_1} < \phi_{t_2} \). In this case \( \partial \Lambda^*/\partial \phi \) might be negative so that lowering \( \tau \) decreases the international HME.

Davis and Weinstein (1999) explain the HME as a magnified impact of a high demand on production. Head and Ries (2001, p. 866) empirically examines the relationship between such a magnified impact and trade barriers. They find that in a two-region model, home market size matters more when trade barriers are lower, which is called the secondary magnification effect (SME) in Head and Mayer (2004). Note that in our model of two countries and four regions, there are three transport costs: \( t_1 \) and \( t_2 \) within countries 1 and 2 respectively, and \( \tau \) for international trade. The SME with respect to those costs are as follows.

\[
\begin{align*}
\frac{\partial \Omega}{\partial \phi_{t_1}} &= -\frac{2\phi_{t_1}}{(1 + \phi_{t_1} - 2\phi_{\tau})^2} < 0, \\
\frac{\partial \Omega}{\partial \phi_{t_2}} &= -\frac{2\phi_{t_2}}{(1 + \phi_{t_2} - 2\phi_{\tau})^2} < 0, \\
\frac{\partial \Omega}{\partial \phi_{\tau}} &= \frac{2(1 + \phi_{t_1})}{(1 + \phi_{t_1} - 2\phi_{\tau})^2} + \frac{2(1 + \phi_{t_2})}{(1 + \phi_{t_2} - 2\phi_{\tau})^2} > 0.
\end{align*}
\]

Comparing with (14), we find that the SME and the infrastructure effect work in different directions with respect to \( t_1 \). Result (i) in the above proposition shows that the latter dominates the former. Meanwhile, the SME and the infrastructure effect work in the same direction with respect to \( t_2 \).

### 3.3 Internationally mobile capital and regional inequality

To obtain the domestic industrial distribution in each country, we substitute (13) into (8) and (10). The equilibrium solution is obtained as follows.

\[
\begin{align*}
\lambda_1^* &= \theta_1 + \Omega_1(2\theta_1 - 1), \\
\lambda_2^* &= \theta_2 + \Omega_2(2\theta_2 - 1),
\end{align*}
\]
where
\[ \Omega_1 = \frac{1}{1 - \phi_{t_1}} \left( \frac{\phi_{\tau}}{\Lambda^*} + \phi_{t_1} - \phi_{\tau} \right), \quad \Omega_2 = \frac{1}{1 - \phi_{t_2}} \left( \frac{\phi_{\tau}}{1 - \Lambda^*} + \phi_{t_2} - \phi_{\tau} \right). \] (16)

**Proposition 3** Under capital mobility across countries, the regional HME in the bigger (smaller) country decreases (increases) with \( \Theta \).

**Proof:** The proposition holds from (16), because \( \Lambda^* \) increases in \( \Theta \). \( \square \)

Behrens et al. (2007, P.1829) find that the economic geography of a country \( i \) depends on the domestic cost \( t_i \) as well as international cost \( \tau \), but *not* on the domestic cost \( t_j \) of the other country. In the present, it is only true when capital is internationally immobile by (8) and (10). In contrast, when capital can move across countries, \( \lambda_1 \) and \( \lambda_2 \) evidently depend on both \( t_1 \) and \( t_2 \) in (15). Intuitively, when the domestic transport cost in a country changes, it changes the incentives of production, causing capital to move not only domestically but also internationally. This further leads to changes of industrial location in both countries.

We next explore the details by examining how the various transport costs impact on the international and domestic industrial location. We find:

**Proposition 4** (i) \( \Omega_1 \) increases in \( \phi_{t_2} \);
(ii) \( \Omega_1 \) increases in \( \phi_{t_1} \) if \( \phi_{\tau} \) and \( \phi_{t_2} \) satisfy
\[ \Theta[(1 + \phi_{t_2})(1 + \phi_{\tau}) - 4\phi_{\tau}^2]^2 \geq \phi_{\tau}(1 + \phi_{t_2} - 2\phi_{\tau})(3 + \phi_{\tau})(1 + \phi_{t_2}) - 8\phi_{\tau}^2]. \] (17)
Otherwise, there exists \( \tilde{\phi}_{t_1} \in (\tau, 1) \) such that \( \Omega_1 \) increases in \( \phi_{t_1} \in [\tau, \tilde{\phi}_{t_1}] \) and decreases in \( \phi_{t_1} \in [\tilde{\phi}_{t_1}, 1] \);
(iii) The relationship between \( \Omega_1 \) and \( \tau \) is related to the relative magnitude of the domestic transport costs \( t_1 \) and \( t_2 \) in the two countries. If \( t_1 = t_2 \), then \( \Omega_1 \) increases in \( \phi_{\tau} \) if and only if \( \phi_{\tau} < \tilde{\phi}_{\tau} \), where
\[ \tilde{\phi}_{\tau} = \frac{1 - \Theta}{\Theta + \sqrt{2\Theta - 1}} \frac{1 + \phi_{t_1}}{2}; \] (18)
(iv) \( \Omega_2 \) increases in \( \phi_{t_1} \);
(v) \( \Omega_2 \) increases in \( \phi_{t_2} \) if \( \phi_{t_1} \) and \( \phi_{\tau} \) satisfy
\[ (1 - \Theta)[(1 + \phi_{t_1})(1 + \phi_{\tau}) - 4\phi_{\tau}^2]^2 \geq \phi_{\tau}(1 + \phi_{t_1} - 2\phi_{\tau})(3 + \phi_{\tau})(1 + \phi_{t_1}) - 8\phi_{\tau}^2]. \] (19)
Otherwise, there exists \( \tilde{\phi}_{t_2} \in (\phi_\tau, 1) \) such that \( \Omega_2 \) increases in \( \phi_{t_2} \in [\tau, \tilde{t}_2) \) and decreases in \( t_1 \in [\tilde{\phi}_{t_2}, 1] \);

(vi) The relationship between \( \Omega_2 \) and \( \tau \) is related to the relative magnitude of the domestic transport costs \( t_1 \) and \( t_2 \) in the two countries. If \( t_1 = t_2 \), then \( \Omega_2 \) increases in \( \phi_\tau \).

**Proof:** See the Appendix.

Now we provide some intuition to the detailed results above. First, in the literature it has been shown that the domestic geographies in the two countries are related if there is a gated region with a lower transport cost than other regions (Behrens et al. 2006a), and/or there are density economies with decreasing marginal transport cost (Behrens et al., 2006b). As a complement to their analysis, our results (i) and (iv) say that reducing the transport cost in one country increases the inequality in the other country, under capital mobility across country borders. It arises because this action attracts more firms away from the smaller region in the latter country.

Next, while the inequality in one country has a simple relationship with the transport costs of the other country, parts (ii) and (v) show that the relationship with its own transport cost is more complicated. The reason is, on the one hand, the foreign transport costs only affect the national share \( \Lambda^* \), which has a monotonic relation with the interregional inequality given by (iii) of Proposition 1; On the other hand, the domestic transport costs have direct impacts on the interregional inequality, and this effect works in the opposite direction to that of the increased national manufacturing share.

To gain more details on this complicated interaction, we now further examine (19).\(^3\)

When the trade cost \( \tau \) is large enough such that \( \phi_\tau = 0 \), then (19) is always true. Therefore, \( \Omega_2 \) increases in \( \phi_{t_1} \). On the other hand, if \( t_2 \) is small such that \( \phi_{t_2} = 1 \), then (17) degenerates to \( 4\phi_\tau^2 \Theta + \phi_\tau (4\Theta - 1) + \Theta - 1 \leq 0 \), or

\[
\phi_\tau \leq \tilde{\phi}_{t_2} \equiv \frac{\sqrt{1 + 8\Theta} + 1 - 4\Theta}{8}.
\]

Therefore, \( \Omega_2 \) increases in \( \phi_{t_2} \) if \( \phi_{t_2} < \tilde{\phi}_{t_2} \) and decreases if \( \phi_{t_2} > \tilde{\phi}_{t_2} \). Similar conclusion holds when \( \phi_\tau \) or \( \Theta \) is large. We thus obtain an inverted U-curve.

---

\(^3\) If we treat the other country as the rest of the world, then analyzing the smaller country is better, since no country is larger than one half of the whole population in the real world. Therefore, we are more interested in (19) than in (17).
The inverted U-shape curve is often called the Williamson hypothesis, or the Kuznet hypothesis in development economics. Such an inverted U-curve has been shown within a one-country-two-region model, by incorporating urban costs (Tabuchi, 1998), or workers’ immobility (Puga, 1999), or agricultural transport cost (Fujita, et al. 1999; Picard and Zeng, 2005), and within a three-location model in Krugman and Livas Elizondo (1996). However, in the two-country-four-region model of Monfort and Nicolini (2000) and the three-location model of Paluzie (2001), the authors find that regional inequality increases monotonically as international economic integration progresses. In contrast to all the above, our results (ii), (iii) and (iv) show that, when production factors are mobile and heterogeneous domestic transport costs are allowed in the two countries, it is possible to obtain both the monotonically increasing form and the inverted U-curve, depending on the parameter values.

Most existing literature also assumes

\[ \Theta = \frac{1}{2}, \quad t_1 = t_2 \equiv t. \]  (20)

In this case, \( \Lambda = 1/2 \) and \( \Omega_1 = (\phi_t + \phi_\tau)/(1 - \phi_t) \). Therefore,

\[ \frac{\partial \Omega_1}{\partial \phi_t} = \frac{1 + \phi_\tau}{(1 - \phi_t)^2} > 0, \quad \frac{\partial \Omega_1}{\partial \phi_\tau} = \frac{1}{1 - \phi_t} > 0 \]  (21)

always hold. In contrast, our results of (ii) and (iii) reveal a more complicated relationship between \( \Omega_1 \) and \( t_1 \), (also between \( \Omega_1 \) and \( \tau \)) in general situations. While (21) is consistent with the conclusion of Monfort and Nicolini (2000), this limited result comes from two respects of assumption (20). First, the assumption of identical domestic transport costs \( (t_1 = t_2 \equiv t) \) automatically implies that improvement in one country is accompanied by improvement in the other country; Second, the assumption of no size difference in the two countries implies no relative HME between them. Thus \( \phi_\tau \leq \phi_{t_i} < \tilde{\phi}_\tau \) always holds when \( \Theta = 1/2 \).

Finally, (iii) and (iv) show that reducing the international trade cost \( \tau \) have different impacts on the regional inequalities in the two countries with \( t_1 = t_2 \). The interregional

---

4Empirical studies on China (Fujita and Hu, 2001), Mexico (Sánchez-Reaza and Rodríguez-Pose, 2002), Greece (Petrakos and Saratsis, 2000) and the EU–15 (Davies and Hallet, 2002) support the hypothesis. Williamson (1965) argues that catching-up countries enjoying a high national growth rate are often associated with rising regional disparities. As growth proceeds, however, regional disparities are hypothesized to fall.
inequality in Country 1 behaves as an inverted U-curve while the interregional inequality in Country 2 is monotonically increasing. Particularly, when \( \Theta \) is large such that \( \phi_r > \tilde{\phi}_r \), \( \Omega_1 \) is decreasing in \( \phi_r \) but \( \Omega_2 \) is increasing in \( \phi_r \). Thus the inequality in the small country becomes more severe than in the large country. This conclusion is consistent with (ii) of Proposition 2 and (iii) of Proposition 1. It also suggests that as industries agglomerate in the larger country, the larger region in it is not the single winner because the interregional inequality does not increase in the larger country. Unfortunately, the situation in the remote region of the smaller country is just the opposite. This result cannot be obtained either with a one-country-two-region model, or with a symmetric two-country-four-region model, as in the existing literature.

4 Conclusions

Based on a footloose-capital model of two countries and four regions, this paper examined the interaction of interregional and international inequalities in the presence of domestic transport costs and international trade costs.

Different from the literature, we assumed countries as well as regions to be asymmetric, and allowed capital to move across countries. These turned to be important because they generated richer and more realistic results than those found in the existing literature. We find that the national manufacturing share is jointly determined by all transport costs, domestic and international. Better infrastructure and governance increase production incentives and attract more firms. A larger national manufacturing share leads to a smaller interregional inequality.

Consequently, the interregional inequality in a country is related to not only its own domestic transport cost, but also the domestic transport cost in the other country. Furthermore, interregional inequality can exhibit a monotonically increasing relation or an inverted-U shape under globalization, depending on each country’s infrastructure and governance efficiency, as well as the international trade cost, even though the present model employed the standard centrifugal force (the immobile consumers), as in Monfort and Nicolini (2000) and Paluzie (2001). These complement the existing literature and hopefully stimulate more research interests on related issues.

Acknowledgement: We wish to thank Hajime Takatsuka and seminar participants at Kobe University for helpful comments. The usual disclaimer applies.
Appendix

(i) This is simply because

\[
\frac{\partial \Omega}{\partial \phi_{t_2}} = \frac{2\Theta \phi_r^2}{(1 - \phi_{t_1})(1 + \phi_{t_2} - 2\phi_r)^2} \frac{1}{(\Lambda^*)^2} > 0.
\]

(ii) The partial derivative of \( \Omega \) with respect to \( \phi_{t_1} \) is

\[
\frac{\partial \Omega}{\partial \phi_{t_1}} = \Theta \frac{(1 - \phi_{t_1})^2}{(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2} - 2(1 + \phi_{t_2} - 2\phi_r)\phi_r
\]

\[
\times \left\{ \Theta[(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2] - (1 - \phi_{t_1})(1 + \phi_{t_2} - 2\phi_r)\phi_r \right\}
\]

\[
\times \left[ (1 - \phi_{t_1})^2 \Theta[(1 + \phi_{t_2})(1 + \phi_{t_2} - 4\phi_r^2) - (1 + \phi_{t_2} - 2\phi_r)\phi_r] \right.
\]

\[
- \phi_r[(1 + \phi_{t_2} - 2\phi_r)^2[4 - (1 - \phi_{t_1} - \phi_r)^2 - 3\phi_r^2]
\]

\[
- 2\phi_r^2(\phi_{t_1} - \phi_{t_2})(1 + \phi_{t_2} - 2\phi_r)(3 - \phi_{t_1} - 2\phi_r) \}.
\]

The fraction outside the curly braces is positive because of (12). Let \( \Omega_{10} \) denote the terms inside the curly braces. Then we have

\[
\frac{\partial \Omega_{10}}{\partial \phi_{t_1}} = \frac{2(1 + \phi_{t_2})}{(1 - \phi_{t_1})^2}\left[ \Theta[\Theta(1 + \phi_{t_2})(1 + \phi_{t_2} - 4\phi_r^2) - (1 + \phi_{t_2} - 2\phi_r)\phi_r] \right.
\]

\[
\times \left\{ \Theta[(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2] - (1 - \phi_{t_1})(1 + \phi_{t_2} - 2\phi_r)\phi_r \right\}
\]

\[
- \phi_r[(1 + \phi_{t_2} - 2\phi_r)^2[4 - (1 - \phi_{t_1} - \phi_r)^2 - 3\phi_r^2]
\]

\[
- 2\phi_r^2(\phi_{t_1} - \phi_{t_2})(1 + \phi_{t_2} - 2\phi_r)(3 - \phi_{t_1} - 2\phi_r) \}.
\]

where the first inequality is from (12), and the second inequality is from the assumption of \( t_1 \leq \tau \) and \( t_2 \leq \tau \). Our conclusion is true because

\[
\Omega_{10}\bigg|_{\phi_{t_1}=1} = 4(1 - \phi_r)(1 + \phi_{t_2} - 2\phi_r)[\Theta(1 + \phi_{t_2} - 2\phi_r) - (1 + \phi_{t_2} - 2\phi_r)\phi_r]
\]

\[
> 4(1 - \phi_r)(1 + \phi_{t_2} - 2\phi_r)
\]

\[
\times \left\{ \frac{1}{2}\Theta[(1 + \phi_{t_2})(1 + \phi_{t_2} - 4\phi_r^2) - (1 + \phi_{t_2} - 2\phi_r)\phi_r] \right\}
\]

\[
> 0
\]
hold and

\[ \Omega_{10}\big|_{\phi_t = \phi_r} = (1 - \phi_r)\{\Theta[(1 + \phi_t)(1 + \phi_r) - 4\phi_r^2] - (1 + \phi_t - 2\phi_r)\phi_r[(3 + \phi_r)(1 + \phi_t) - 8\phi_r^2]\} \]

is nonnegative if and only if (17) holds.

(iii) The partial derivative of \( \Omega_1 \) with respect to \( \phi_r \) is

\[
\frac{\partial \Omega_1}{\partial \phi_r} = \frac{\Theta(1 + \phi_t - 2\phi_r)^2[(1 + \phi_t^2)^2 + 4\phi_r^2 - \Theta(1 + \phi_t + 2\phi_r)^2]}{(1 - \phi_t^2)[\Theta][(1 + \phi_t^2)(1 + \phi_t - 4\phi_r^2) - 2(1 + \phi_t - 2\phi_r)\phi_r^2]} - \frac{4\Theta(\phi_t - \phi_r)\phi_r}{(1 - \phi_t^2)[\Theta][(1 + \phi_t^2)(1 + \phi_t - 4\phi_r^2) - 2(1 + \phi_t - 2\phi_r)\phi_r^2]^2}.
\]

The second term has a factor of \( \phi_t - \phi_r \), which is the differential of the trade freeness in the two countries. The first term is positive if \( \phi_r < \tilde{\phi}_r \) of (18) and negative if \( \phi_r > \tilde{\phi}_r \).

(iv) This is simply because

\[
\frac{\partial \Omega_2}{\partial \phi_t} = \frac{2(1 - \Theta)\phi_r^2}{(1 - \phi_t^2)(1 + \phi_t - 2\phi_r)^2} \frac{1}{(1 - \Lambda_s^2)^2} > 0.
\]

(v) The partial derivative of \( \Omega_2 \) with respect to \( \phi_t \) is

\[
\frac{\partial \Omega_2}{\partial \phi_t} = \frac{1 - \Theta}{(1 - \phi_t^2)^2[(1 - \Theta)(1 + \phi_t)(1 + \phi_r) - 4\phi_r^2] - 2(1 + \phi_t - 2\phi_r)\phi_r^2} \times \left\{ (1 - \Theta)(1 - \phi_r)[(1 + \phi_t)(1 + \phi_t - 4\phi_r^2)] - \phi_r(1 + \phi_t - 2\phi_r)^2[4 - (1 - \phi_t - \phi_r)^2 - 3\phi_r^2] - 2\phi_r^2(\phi_t - \phi_t)(1 + \phi_t - 2\phi_r)(3 - \phi_t - 2\phi_r) \right\}.
\]

The fraction outside the curly braces is positive because of (12). Let \( \Omega_{20} \) denote the terms inside the curly braces. Then we have

\[
\frac{\partial \Omega_{20}}{\partial \phi_t} = 2(1 + \phi_t^2)[(1 - \Theta)(1 - \phi_r)[(1 + \phi_t)(1 + \phi_r^2) - 4\phi_r^2] - (1 - \phi_t)(1 + \phi_t - 2\phi_r)\phi_r] > 2(1 + \phi_t^2)[2(1 - \phi_r)(1 + \phi_t - 2\phi_r)\phi_r - (1 - \phi_t)(1 + \phi_t - 2\phi_r)\phi_r] \]
where the first inequality is from (12), and the second inequality is from the assumption of \( t_1 \leq \tau \) and \( t_2 \leq \tau \). Our conclusion is true because

\[
\Omega_{20}\big|_{\phi_{t_2} = 1} = 4(1 - \phi_r)(1 + \phi_{t_1} - 2\phi_r)[(1 - \Theta)(1 + \phi_{t_2} - 2\phi_r) - (1 + \phi_{t_1} - 2\phi_r)\phi_r]
\]

\[
> 4(1 - \phi_r)(1 + \phi_{t_1} - 2\phi_r)
\]

\[
\times \left\{ \frac{1 - \Theta}{2}[(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2] - (1 + \phi_{t_1} - 2\phi_r)\phi_r \right\}
\]

\[
> 0
\]

always hold and

\[
\Omega_{20}\big|_{\phi_{t_2} = \phi_r} = (1 - \phi_r)\{(1 - \Theta)[(1 + \phi_{t_1})(1 + \phi_r) - 4\phi_r^2]^2
\]

\[
- (1 + \phi_{t_1} - 2\phi_r)\phi_r[(3 + \phi_r)(1 + \phi_{t_1}) - 8\phi_r^2]\}
\]

which is nonnegative if and only if (19) holds.

(vi) The partial derivative of \( \Omega_2 \) with respect to \( \tau \) is

\[
\frac{\partial \Omega_2}{\partial \phi_r} = \frac{(1 - \Theta)(1 + \phi_{t_1} - 2\phi_r)^2[(1 + \phi_{t_2})^2 + 4\phi_r^2 - (1 - \Theta)(1 + \phi_{t_2} + 2\phi_r)^2]}{(1 - \phi_{t_2})\{(1 - \Theta)[(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2] - 2(1 + \phi_{t_1} - 2\phi_r)\phi_r \}^2}
\]

\[
- \frac{4(1 - \Theta)(\phi_{t_2} - \phi_{t_1})\phi_r \{(\phi_{t_2} - \phi_{t_1})\phi_r \Theta + (1 - \Theta)[(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2]\}}{(1 - \phi_{t_2})\{(1 - \Theta)[(1 + \phi_{t_1})(1 + \phi_{t_2}) - 4\phi_r^2] - 2(1 + \phi_{t_1} - 2\phi_r)\phi_r \}^2}.
\]

The second term has a factor of \( \phi_{t_2} - \phi_{t_1} \), which is the differential of the trade freeness in the two countries. Meanwhile, the first term is always positive according to (12).

References


